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Features of European Arithmetic Programs
CHARLES H. SCHUTTER

The Insatiable Quest: Mathematicking
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THE ARITHMETIC TEACHER

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Some Important Features of European Arithmetic Programs

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AN EXAMINATION OF ARITHMETIC BOOKS from twenty European countries reveals some interesting practices, the most noteworthy of which are listed in this article. (A more complete and detailed analysis of those textbooks will be found in the recently published booklet, *Teaching the Third R, a Comparative Study of American and European Textbooks in Arithmetic*, produced by the Council for Basic Education, 725 Fifteenth Street, N. W., Washington 5, D. C.)

I. Europeans Start Formal Arithmetic Early

As a general rule, all European countries expect the pupils to master all of the addition and subtraction facts, and composition and decomposition of all numbers up to 20, during the first school year. As soon as the child learns the meaning of 6 or 7, for example, he is given practice in adding the smaller numbers to make 6 or 7, and in subtracting the smaller numbers from these larger ones. Multiplication of numbers with products no greater than 20 is taught as repeated addition in a great many European countries during the first school year, and the reverse process of division with dividends 20 or less is usually taught at the same time. Much practice is given in finding numbers which must be added to or subtracted from other numbers to make required sums or remainders, requiring abstract thinking. The following chart contains many such exercises, the question marks indicating that the pupil must find a missing, unseen number by his own mental efforts. Most countries give problems requiring addition of three numbers mentally; some countries give exercises in which these first-year pupils are required to do two different operations in the same problem.

Such arithmetic work no doubt serves to keep the first-year European pupils alert; the variation in the problems and the necessity to think and find required numbers should be stimulating.

During their second school year, European children in general use all of the numbers up to 100. All of the multiplication tables up to and including the 10's are learned, along with the corresponding division facts. Simple fractions such as halves, thirds, and fourths are taught. Much practice is given in adding numbers less than 10 to larger numbers and getting a sum in the next decade of numbers; similar practice is given in subtracting numbers less than 10 and obtaining remainders in a lower decade. The type of mental number practice started in the first year, finding required, unseen numbers, is continued, as shown in Chart 2.

CHART 1

SAMPLES OF PROBLEMS FROM FIRST YEAR EUROPEAN ARITHMETIC BOOKS

France

$7+3+?=15$ $8=3+2+?$ $19-?=14$

Germany

$9+1-7=?$ $9-?=6$ $92+?=99$

Greece

$16-7=?$ $13+4=?$ $12-?=6$

(Europeans use the colon : for the division sign)

$6:2=?$ $10:5=?$ $12:3=?$

$2\times3=?$ $3\times5=?$ $8\times2=?$

Holland

$6+?=9$ $9-4-3=?$ $6+3-5=?$

and problems requiring the pupil to supply addends of his own choosing:

$?+?=7$ $?+?=9$

Hungary

$10-?=6$ $17-7-2=?$ $17=9+?$

Poland

$5+3+2=?$ $1+2+3+?=10$ $16-13=?$

$3\times2+9=?$ $4\times5-6=?$ $25-20+80=?$

(Two processes in same problem)

Jugoslavia

$3+?=8$ $6+5+?=19$ $10+10-?=13$

$3\times5+2=?$ $6=2\times?$ $15:3=?$

CHART 2

TYPES OF WORK FOUND IN SECOND YEAR EUROPEAN ARITHMETICS

France

$27+?=35$ $91-?=88$ $54=?\times6$ $72=8\times?$

Germany

$22+?=40$ $63+?=80$ $47-2+8=?$ $94-7-9=?$

Greece

$26+18=?$ Illustrated as $26+10+8$, for mental computation.

$83-25=?$ Illustrated as $83-20-5$, to be done mentally.

Multiplication and division: $9\times7=?$ $45:5=?$ $56:8=?$

Holland

$56-4-10=?$ $91-14=?$ $74=9\times8+?$ $6\times10+?=100$

Supplying factors unaided: $?\times?=27$ $?\times?=42$

Poland

$88-9+6=?$ $100-50-20=?$ $48=6\times?$ $54:?=9$

$5\times8+2\times8=?\times8$ $7\times4+7\times4=7\times?$

Sweden

$54+33+10=?$ $197-181=?$ $329-6-2=?$ $84-8-6=?$

Switzerland

$9\times5+8=?$ $8\times3-?=18$ $70-?=30$ $?\times4=28$

CHART 3

SAMPLES OF THIRD YEAR EUROPEAN ARITHMETIC WORK

France

Oral Exercises:)

730+60=? 4780+40=? 860−34=? 1000=200+?

(Written Work)

6510	7819	1752 : 4	5664 : 6	18909 : 9
−4878	−....?			
?	5308	78×23	609×34	4065×23

Germany

(Oral Exercises)

800−300−300=? 700−300+500=? 653−8−5=?

700−?=500 500=900−? 200=900−?

Multiplication of type like 6×32 taken as 6×30 plus 6×2, or 180+12, or 192. Exercises like 8×23, 7×27, etc., to be done the same way, mentally.

210=?×30 420=?×70 105=?×15

Greece

(Oral)

24×10	49×10	70×10	3×100	8×100
480 : 10	950 : 10	750 : 10	900 : 100	700 : 100

Holland

Count forward by 80: 200, 280, 360, etc, up to 1000.
Count backward by 60: 900, 840, 780, etc., down to 600.

Hungary

(Oral)

600+?=900 86+?=1000 3×200+70=? 7×20+16=?

Poland

(Oral)

200:50 420:70 1600:40 4000:500 6300:700

2×4000 shown to be same as 20×400 or 200×40 or 2000×4.

Jugoslavia

(Oral)

Subtract from 970 the numbers 170, 270, 370 · · · successively.
Add 273 the numbers 100, 200, 300, 700, and 400, successively.

In their third school year, Europeans deal with numbers as high as a thousand or more. Addition with carrying and subtraction with borrowing are taken up, although some few countries take these processes up as early as the second school year. Facility in handling large numbers is developed. Pupils are expected to do problems like 8+340, or 20+748, or 962−5 mentally. Chart 3 shows some of the problems requiring pupils to think in terms of large numbers.

II. European Children Are Expected to Do a Great Deal of Their Arithmetic Mentally

Written work seems to be required only when the problem cannot be solved by hard thinking, concentrating, and remembering interim results. Likewise, large numbers are expected to be handled mentally, without recourse to writing. The illustrations in Chart 3 are examples of this emphasis on keeping large numbers in the mind.

In doing long division, this skill in handling large numbers mentally and in getting an over-all idea of the sizes of numbers is helpful to the European students because it enables them to obtain correct trial quotients by using the entire divisor (or a close approximation to it) for finding the quotient. For example, a fourth-grade Polish arithmetic book shows the student who has the problem of dividing 987 by 24 that he must think how many times 24 is contained in 98, as a first step. He tries 3, but 3×24 is only 72, so he tries 4. Since 4×24 makes 96, which is less than 98, the first figure in the quotient will therefore be 4. The student is not told to first see how many times the 2 in the divisor 24 is contained in the 9 of the dividend 987, which would obscure the entire problem and weaken the student's number concept and grasp.

The challenge to the student's thinking power is evident in all of the work illustrated in the three charts. Alertness is encouraged by the problems requiring several successive additions or subtractions, and by the necessity of remembering results already obtained for the final solution or result.

Most European arithmetic books provide interesting practice for the students by the use of "Magic Squares." These help the students to find the required number to make a certain sum. Such amusing devices were found in arithmetic books from Austria, Greece, Italy, Russia, and other countries.

Some arithmetics showed problems with erroneous answers, and the pupils were required to find the errors.

Many European arithmetics appeal to the child's natural curiosity and interest in puzzles. A third grade Hungarian arithmetic presents this set of problems in which the pupil must decide which processes must be used to give the stated result—the processes indicated by the question marks:

$$2?4?5=11$$

$$2?4?5=30$$

$$2?4?5=\ 1$$

$$2?4?5=13$$

An example of a device to encourage concentration of effort and persistence, and which should interest the pupil at the same time, is the use of a chain of problems each of which has several parts, and in which the sum of the answers to the separate problems in the chain must equal a result stated in the textbook. An arithmetic for the third school year, used in Germany, presents a chain of five problems, one of the five being this:

$$\begin{array}{r} 8\times16 \\ +72 \\ :\ 4 \\ \times\ 6 \\ -63 \\ +\ 3 \\ :80 \\ \hline \end{array}$$

The answer, (3), was to be added to the answers from the four other similar problems in the chain, and give 95 for the sum of the five. The student could thus check his own work and locate errors, besides getting enjoyment from the solving of the problem.

III. European Arithmetic Programs Are Rigorous

It should be borne in mind that all of the arithmetic work cited in this article is expected to be done by the average European child, not by a selected group. The selection process does not begin in the European schools until after the sixth school year, approximately. All of the problems which have been illustrated are in the program for the average European child.

The sample problems in the charts are evidence of the rigor in European programs. However, these have been taken from only the first three years of the arithmetic program. Following are some examples from other grades.

A third-grade Rumanian arithmetic gives the problems 254×323, and 942×654.

A Dutch arithmetic for the fifth grade presents this challenge:

$$\frac{(\frac{1}{2}+0.3+0.57+\frac{7}{8})\times51.3}{0.449}$$

equals how much?

An Irish arithmetic for the sixth school year gives:

$$\frac{\frac{3}{8}}{\frac{1}{4} \text{ of } \frac{1}{6}} \text{ divided by } \frac{1\frac{1}{4}}{20}$$

equals how much?

Arithmetics used in England and Ireland show rigorous and complicated work in fractions, especially in denominate numbers used in the English system of money and measurements. An English arithmetic for the sixth school year gives this problem:

"Express 17 shillings, $2\frac{1}{2}$ pence as a decimal part of a pound, to two places." This difficult problem was to be done mentally, according to the textbook, and the student was to obtain 0.8604 pound, or 0.86 for two places.

IV. European Arithmetic Textbooks Give Adequate and Sound Explanations of Processes and Methods

Pupils are not told or encouraged to do things mechanically before understanding the why and the wherefore of the processes. In many cases, alternate procedures or methods are shown, and the student is allowed to use the method which he prefers. Thus, in dividing by a fraction, the common denominator method and the "invert and multiply" methods are usually shown together in the European arithmetic book. Place value is clearly explained. Carrying and borrowing are thoroughly illustrated and explained. Multiplication as a short cut for addition is pointed out.

Work in decimals is closely related to the money and measurement systems of the various countries. (The English system of measurements, however, does not lend itself readily to decimals, with the result that English and Irish arithmetics introduce decimals later than the countries on the continent where the Metric System prevails.) Work in precentage is closely interwoven with work in decimals throughout European arithmetic programs.

Story or word problems usually deal with incidents or events commonly known throughout the particular country or communities and are practical in nature. However, some problems appeal to the imagination and to the intellectual curiosity of the student. One can say with assurance that European arithmetic teaching and learning is based on meaning and understanding, and that it makes full use of the knowledge of child psychology.

V. European Mathematics Programs Introduce Geometrical Concepts Early and Develop Them Gradually from Year to Year

For example, in Belgium, France, and Italy some simple geometric figures are introduced and discussed in the second or third school year, and geometric concepts are taken up along with the regular work in numbers. A Belgian arithmetic for seven-year-old pupils illustrates the properties of the square, the rectangle, and the triangle. Further knowledge of these figures is given in the third school year, and areas and perimeters are found; right triangles and isosceles triangles are illustrated and discussed. In the fourth school year, the Belgian pupil learns about parallels, perpendiculars, diagonals, and medians of triangles; equilateral triangles are discussed, and acute and obtuse angles are defined and illustrated. In the fifth school year, formulas for areas and volumes of figures such as the circle, cone, cylinder, prism, and pyramid are given.

The early introduction of geometric concepts, and their gradual and continuous development from about the second or third school year onward, is an outstanding feature of the mathematics programs of many European countries. (The geometry and arithmetic work is usually presented in the same textbooks, although some countries have different books for the geometry.)

VI. Miscellaneous Notes

In practically all European countries, the comma is used instead of the decimal point. Also, a decimal fraction which is less than 1 is invariably written with a 0 in the unit's place: thus, six-tenths is written as 0,6 in the European style.

The colon : is used instead of the sign ÷ to indicate division.

Common fractions with the denominator 100 seem to be preferred to the decimal form in dealing with percentage calculations. Thus, to find $3\frac{1}{2}\%$ of 800, most Europeans would multiply $\frac{7}{200}\times 800$, instead of multiplying 0.035×800.

Counting backwards is given much practice in European arithmetic.

In logarithms (given in the eighth year of some programs) the form $9.8660-10$ was not found; instead, the equivalent $\bar{1}.8660$ was employed.

The school years in which certain topics are taken up are shown below.

CHART 4

GRADE PLACEMENT OF TOPICS IN ARITHMETIC PROGRAMS

Topic	Grade or Year of Introduction							
	England	France	Germany	Greece	Holland	Poland	Sweden	United States
Multiplication and Division Tables up to the 10's.	3 (to 12's)	2	2	2	2	2	2	4
Multiplication with Carrying, 1-digit multipliers, like 346×4.	3	2	3	3	3	3	3	3A–4B
Multiplication with 2-digit mulitpliers; like 347×54.	3	3	4	4	4	4	4	4A–5B
Addition with Carrying and Subtraction with Borrowing.	3	2	3	3	3	3	4	3A–4B
Division by 1-digit Numbers, with carrying, like $6\overline{)492}$.	3	3	3	3	3	3	3	4
Division by 2- or 3-digit Numbers.	3	3	4	4	4	4	4	5
Common Fractions:								
Adding and Subtracting with the same Denominators:	3	5	3	5	4	4	5	5
Adding and Subtracting with prime Denominators; e.g. with 3rd and 5ths.	5	6	6	5	5	5	6	5–6
Multiplication and Division of Integers by Fractions; like $6\times\frac{2}{3}$, $8\div\frac{3}{4}$.	5	5	5	5	5	6	5	6
Decimals:								
Multiplying integers by decimals, like 8×0.63	6	4	4	5	5	5	5	6
Multiplying and dividing decimals by decimals; e.g., 6.3×0.7; $6.24\div 0.36$.	6	4	6	5	6	6	6	6
Percentage	6	5	6	6	6	6	6	7

The preceding chart showing grade placement of topics in European arithmetic programs is based upon examination of textbooks currently in use throughout Europe. These placement figures are the ones most generally fitting the arithmetic programs; no doubt, in some parts of Europe, as in some parts of our own country, mathematics programs and curricula may vary from the general patterns. However, the placement of a given topic would not be likely to vary significantly from that shown in the chart.

While it is not the purpose of this article to compare the arithmetic programs of the United States with those of Europe, it seemed advisable to supply data from the United States so that our readers would have a better comprehension of the material and discussion presented in this article, than they would have had without some brief reference to our own placement of topics in arithmetic.

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EDITOR'S NOTE. What is our answer to the data presented by Dr. Schutter in Chart 4 where he compares grade placement of topics in several European countries with that in the United States? What values do we want from our schools at the primary and intermediate grade levels? These questions will be answered differently by different people within any gathering of school people. It is important that they be discussed and viewed openly and intelligently. Certainly we can teach more arithmetic in the primary grades because we did so more than a generation ago. Are young children prepared and able to learn more? Studies of child interests and abilities say yes. Shall we then teach more arithmetic in grades one and two and also introduce geometry much earlier than we now do? We probably will do so. Dr. Schutter points out that in Europe it is not an arithmetic of drill and memory alone but is one of thinking and understanding. We have been emphasizing understanding in this country but not of the type and on a scale comparable with Europe. Their arithmetic is more "Mathematical" and perhaps less of the "counting house" variety. Shall we move closer to Europe?

We are grateful to Dr. Schutter not only for presenting the grade-level of topics but also for illuminating them with illustrations from the several countries. Teachers may wish to use some of the sample items which call for a non-rote level of thinking and discovery. Will teachers and school people oppose the earlier mastery of topics? Are our teachers educated so they can and will face the issues in deciding what should be done about arithmetic in the United States? It is interesting to note that only one state, Louisiana, now requires six semester hours of mathematical training for newly trained elementary school teachers. Is that a requirement toward which other states should progress?

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The Insatiable Quest: Mathematicking[1]

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THE WORDS OF THE TITLE for this paper are freighted with meaning. The first three words, THE INSATIABLE QUEST, describe the basic method of learning; they apply to areas of the curriculum. The last word, MATHEMATICKING, defines the process of achieving efficient and intelligent behavior with regard for size, order, position, and amount. It is the process through which the pupil's CONQUEST of an education of mathematics should evolve.

It is interesting to note related QUEST terms—BEQUEST, REQUEST, INQUEST, and CONQUEST. BEQUEST, INQUEST, and REQUEST may retard a pupil's achievement of a high level of mathematical competency. Perhaps we find a considerable lack of success in many programs because the educator's attempt is to develop mathematics as a BEQUEST, i.e., as if it were a body of knowledge handed down—to be transmitted—and this transmission is often conceived to be an osmotic process. When the program of learning fails, sometimes an INQUEST is held, and the reasons for failure are attacked. The next step may be the solution which becomes a REQUEST that students take more work in less time, or that they work harder, or that they do more of the same type of thing that brought about what was *conceived* in the BEQUEST and *executed* in the INQUEST and REQUEST! In other words, the REQUEST is that of a single unhyphenated word which may be defined as "to ask for" or "to demand." If we hyphenate the word, we can coin the word, RE-QUEST, which may mean a return QUEST in which the QUEST becomes a search to see what one didn't see before. The RE-QUEST may lead to CONQUEST of mathematics.

Let us apply these points to a specific problem. The one selected for this paper is a simple one that centers around multiplication and division of decimal fractions and common fractions. In order to determine the concepts that are held by students concerning these areas, an exercise of eight simple questions concerning the nature of the answers to examples was developed. These statements were clothed in simple words designed to be interpreted by pupils of grade six level. With each of the eight questions, the student was asked to state his ideas and was asked to refrain from numerical computation. The purpose of the exercise was to determine what the pupil thought about the nature of the answers. The exercise was given to 243 college students, 167 pupils in grades seven and eight, 220 pupils in grade six, and one class of 27 children in a combined grade four-five class. The classes of children in grades six, seven, and eight were groups that were considered average or above average. The major portion of the college students were in teacher-education courses, and thus a process of selection had operated. Some of the students had taken courses in college mathematics, but none of them had more than started their course in methods for teaching mathematics in the elementary school.

Lest there be misinterpretation with regard for FIGURES, a foreward is in order. After all, this is a group for whom FIGURES count! The findings that are reported are gathered from several states of our 50 United States. Since it was stated that names of places, teachers, cities, etc., would not be revealed, the findings are reported as general findings. It may be added that California didn't stand out as a bright and shining star; neither did it stand as a star of low magnitude. No attempt is made to indi-

[1] A paper presented at the Elementary Section, N.C.T.M. meeting in Dallas, Texas, April, 1959.

cate that any special vicinity is not "way out front" and that your classes have been led to an INSATIABLE QUEST so that CONQUEST of mathematical concepts is totally different than any findings presented here! Let us look at some of the questions and analyze the nature of the CONQUEST of mathematical relationships.

Test Questions and Responses

The directions for the test are given below.

"Do Not Work These Examples. You will be asked simple questions concerning the answers. For each question, underline the answer that you think is true."

QUESTION 1

Susan said, "The answer to 4348 divided by .25 will be larger than 4348." What Susan said is

(a) true. (b) false. (c) partly true.
(d) partly false.

Question 7 was paired with question 1 to check the interpretation of division of decimal fractions.

QUESTION 7

Betty said, "The answer to any number except zero divided by .85 will be larger than that number." What Betty said is

(a) always true. (b) sometimes true.
(c) sometimes false. (d) always false.

The percentages of *incorrect* response were as follows:

Group	Que. 1	Que. 7
Combined population of college classes	65	63
Combined population of grades seven and eight	83	71
Combined population of grade six classes	84	75
Grade four-five combination class	78	63

Questions 4 and 8 were paired to check interpretation of division of common fractions.

QUESTION 4

The answer to 7735 divided by $\frac{9}{10}$ will be

(a) larger than 7735. (b) smaller than 7735.
(c) equal to 7735.

QUESTION 8

Martha said, "The answer to any number except zero divided by $\frac{15}{16}$ will be smaller than that number." What Martha said is

(a) always true. (b) sometimes true.
(c) sometimes false. (d) always false.

The percentages of *incorrect* response were as follows:

Group	Que. 4	Que. 8
Combined population of college classes	64	67
Combined population of grades seven and eight	78	78
Combined population of grade six classes	72	80
Grade four-five combination class	78	78

Questions 2 and 6 were paired to determine pupil interpretation of multiplication of common fractions. Questions 3 and 5 were paired for a similar treatment of multiplication of decimal fractions. Although the percentages of error were lower for multiplication than for division, only questions 2 and 3 produced less than 50 per cent error for any of the groups tested in the study.

Using a Newer Approach

Are the concepts extremely difficult to comprehend? Are they teachable to young children? In order to delve into these problems, two classes in which the multiplication and division of common and decimal fractions had not been taught were used as experimental groups. In one class, a grade six group of 38 children, there had been an introduction to multiplication of common fractions, but the introduction as indicated by results on the pretest, did not interfere with the study. The other group was a mixed class of 27 children in grades four and five. The group was above average. Each was pretested, and teaching the concepts of division and multiplication of fractions was outlined. Emphasis was placed on building the concepts, and the work was carried on as a brief additional unit during the period allotted for mathematics. The regular work of the class continued since the classes were in a public school in which the program was set up in a definite way. The specific concepts were identified, and the teaching was designed to challenge the learner to sense, discover, and interpret the relationships that underlie the areas of multiplication and division of common and decimal fractions.

Emphasis was placed upon making the relationships meaningful through selection of illustrations that were significant in the experiencing of the children. The concepts grew out of simple, elemental ideas that children used and could interpret, and the generalizations evolved with power behind them.

In the experimental study, the concepts underlying division of decimal and of common fractions were taught as a single unit. In this report, the experiment with grade six will be described as considerable data concerning the study is available.

The teachers were led to identify the basic concepts that underlie the areas of division and multiplication of common and decimal fractions. For example, the main concepts that underlie division of decimal fraction examples of the type used in questions 1 and 7, were identified as

(1) the interpretation of decimal integer and decimal fraction expressions, and
(2) interpretation of the divisor-dividend-quotient relationship that functions when the dividend is constant and the divisor is variable.

First, the teacher challenged the children to sharpen their understanding of decimal integer and decimal fraction expressions. An open-end abacus was used to refine concepts such as the constant and positional value of digits in a numerical expression, meanings of zero, and principles of grouping and regrouping, etc. The extension of the system of decimal integers led to developing the locative function of the decimal point and to interpretation of decimal fraction expressions such as .5, .25, and .09. Comparisons were made of the numerical value of decimal integer and decimal fraction expressions such as 25 and .25, and 9 and .09.

Secondly, ideas of divisor-dividend-quotient relationships were developed. The concept of the relationship that exists when the dividend is constant and the divisor and quotient are variable was isolated and developed. Illustrations without the use of specific number preceded development of examples with specific number. For example, if the dividend shows the distance traveled, the divisor may indicate the speed of travel and the quotient the time it takes for travel.

$$\text{speed}\,)\overline{\text{distance}}$$
$$\text{time}$$

If the distance traveled is constant (dividend), as we *IN*CREASE the speed of travel, we *DE*CREASE the time for travel, and as we *DE*CREASE speed, we *IN*CREASE time. The teacher challenged the children to interpret the divisor-dividend-quotient relationship as it occurred in settings familiar to them. The children illustrated their ideas on large sheets of paper, and the concept of the relationship became very meanful. Refinement of the ideas through using specific numbers followed, and when examples such as 35 divided by .5, 6 divided by $\frac{1}{4}$, etc., were discussed, the children JUST KNEW the answers. Below are a few statements made by the children that indicate the nature of their thinking:

"Our answers come out larger instead of smaller when we divide by a fraction" (larger than the dividend).
"It is interesting to see that dividing can make answers larger than the dividend and that multiplication makes answers smaller than the multiplicand."
"When using a fraction as a divisor, division seems to be multiplication and multiplying by a fraction seems to make multiplication be division."
"Why don't we just change the names of processes around and call them what we are really doing . . . Well, I guess that wouldn't be too good because . . . "

The children were retested two weeks after the pretest. The change in percentages of incorrect response are indicative of the teachability of the concepts. In the grade six class, the chart below shows the gains on the retest compared with the pretest.

As was stated earlier, the experiment was an additional unit and no written computation pertaining to the unit was given to the classes during the experimental period. However, after the children had a vacation period of one week, they were tested on ten simple examples involving computation to test their generalizations. The test revealed mastery with computation that was

similar to the mastery of concepts. For example, 72 divided by .12 and 20 divided by $\frac{1}{4}$ each produced only 8 per cent error. One of the most interesting findings of the study was this evidence regarding a basic principle for practice exercise or drill. *PRACTICE EXERCISES SHOULD PROVIDE OPPORTUNITY TO PRACTICE CORRECT RESPONSE.* The experimental study demonstrated that when sharpness of concept precedes practice, the principle operates in a positive way. In many classrooms, practice provides opportunity for pupils to practice their errors. Here lies one of the deadliest enemies of achievement of mathematical competency.

GRADE 6

PERCENTAGES OF INCORRECT RESPONSE ON PRETEST AND RETEST AND PERCENTAGES OF GAIN

	% Incorrect Response		
Que.	Pretest	Retest	% Gain
1	91.2	5.9	85.3
2	47.1	8.8	38.3
3	67.6	17.6	50.0
4	76.5	2.9	73.6
5	82.3	29.4	52.9
6	73.2	17.6	55.6
7	82.3	8.8	73.5
8	76.5	14.7	61.8

On the basis of the present study, it is suggested that **The Insatiable Quest: Mathematicking** should be pursued by teachers of mathematics. In the pursuit, there should be a careful analysis of the BEQUEST, i.e., the body of knowledge of mathematics. INQUEST should lead to the development of HOW TO TEACH so that the product of teaching is CONQUEST of mathematics by the learner. The teacher, too, may journey into **The Insatiable Quest: Mathematicking.**

EDITOR'S NOTE. One is always shocked when tests scores show a very low level of achievement. In the past score of years this has often been true with measures of understanding of principles in arithmetic. This probably is a fair mirror of the school program and the teaching which have tended to neglect the development of important principles and generalizations. Many times it has been established that pupils tend to achieve that which the school (and teacher) expect. Mrs. Brydegaard's work shows that teaching for recognition and understanding of principles produces results. We need to provide for practice in thinking as well as practice in operations. Our practice should be supervised and at some stages needs guidance by a skillful teacher. "In many classrooms, practice provides opportunity for pupils to practice errors. Here lies one of the deadliest enemies of achievement of mathematical competency."

Locating the Decimal Point in the Quotient

Pupils have learned the algorithm for locating the decimal place in the product, which is to add the decimal places in multiplicand and multiplier, and mark off that number of places in the product.

Division is the opposite of multiplication. Then, to find the number of decimal places in the quotient, subtract decimal places in divisor from decimal places in dividend.* For example, dividing 234.567 by 89.01, the quotient would have one decimal place. Should we want to work out the problem to two decimal places, add enough zeros to the dividend so that there will be a difference of two decimal places between dividend and divisor.

In those cases where the dividend has fewer decimal places than divisor, add zeros until they are equal, and then the quotient will have no decimal places.

This is the algorithm, and is mechanical. It should of course be preceded by the usual meaningful instruction and development.

Contributed by
HERMAN FRIEDMAN
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* Many older teachers will recognize this as the rule they were taught in school. It was common before the era of placing the quotient above the dividend.

Teachers Look at Arithmetic Manuals[1]

MARY FOLSOM
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IN 1931, THE NATIONAL SOCIETY for the Study of Education devoted a yearbook to a report on the textbook, and the authorities conceded that the textbook is the most valuable single piece of instructional equipment available to the teacher.[2] However, the fact that *manual material* has become a large factor in the preparation and selection of arithmetic textbook series has not been similarly recognized nor has any serious attempt been made to evaluate arithmetic manuals in light of teacher needs and usage.

Since 1956, most arithmetic textbook publishers have expended a large amount of time and money in producing a hard-back, combined text-manual known as the "teacher's edition." This text-manual devotes a considerable percentage of the total content area to instructions and explanations for the teacher. It seems reasonable to assume that commercial companies would not consider such an investment unless they were thoroughly convinced that the teacher's edition was a selling point for a series and indirectly, therefore, that teachers found them worthwhile and desirable.

An extensive review of materials dealing with the development of the arithmetic textbook and manual shows that even early books included suggestions for the teacher and the fact that it is emphasized today would lead to the conclusion that authorities in arithmetic consider the teacher's edition a tool of real worth. At the same time, however, there is a feeling on the part of many serious students of the teaching of arithmetic that many of the suggestions given are not practical or even used by teachers. An analysis and evaluation of manual materials by the teachers who use this tool seemed, consequently, to be both logical and timely.

Observation of Teachers Using a Teacher's Edition

In order to make a study of manual material, it was necessary first of all to determine in so far as possible the facts concerning the use of those materials under existing classroom conditions. To determine the prevailing conditions in regard to the use of arithmetic manual materials the observation type of survey was used as a part of the normative-survey method of research.

All data were obtained by direct observation of actual classroom practice. During each observation special care was taken to note how consistently the teacher used (a) the procedures, (b) the enrichment activities, and (c) the concrete and semi-concrete materials suggested in the manual for the specific lesson taught.

An analysis of the observations made in twenty-two sixth grade arithmetic classes showed that:

1. Exactly half of the teachers followed the text page rather than the procedure suggested in the manual for the presentation of the lesson. Even when suggestions were given regarding an introduction to the lesson other than the immediate use of the textbook, only a few teachers made use of them.

2. It was evident that on the whole each pupil in the room was exposed to the same material presented in the same way. The

[1] This article is based on a portion of the writer's doctoral dissertation: FOLSOM, MARY O'HEARN. *A Study of Manual Material in the Field of Arithmetic.* Unpublished Ph.D. Thesis. State University of Iowa, 1958.

[2] NATIONAL SOCIETY FOR THE STUDY OF EDUCATION. *Fiftieth Yearbook. Part II.* "The Teaching of Arithmetic." Chicago: University of Chicago Press, 1951, p. 1.

majority of teachers do not make use of the enrichment activities or differentiations when they are provided. In the few cases where these activities were used, they were presented to the entire class rather than in the manner proposed in the manual.

3. In most of the classrooms, blackboard, chalk, paper, and pencils constituted the bulk of the equipment used to interpret the textbook. The teaching suggestions in the manual used either did not show the teachers how to use concrete or semi-concrete materials to enhance understanding or did not convince them that materials of this type had a place in the classroom.

It is, of course, true that these classroom observations do not constitute either a large or a random sample due to the difficulties involved in visiting a large number of classrooms and in locating school ssytems that had adopted and used a teacher's edition for at least a year prior to the visitation. In fact, only two publishers met the latter qualification. However, I feel that the classes visited were typical of those found in most Iowa towns and representative in that the observations were made in nine different school systems of varying size.

Reasons for Lack of Utilization

The reasons for the failure to make use of much of the material presented in the manual became apparent when an analysis was made of the answers given by these teachers to a questionnaire left with them at the end of the classroom observation. While it is true that teachers are not usually familiar with a large number of different manuals, it is my belief that they can make valid statements regarding the features of the manual they are using if the questions asked cause them to stop and weigh the advantages and shortcomings of the suggestions given in their manual.

It is well to keep in mind that the per cents quoted below are based on a small sample and that only two manuals are considered. The latter fact accounts in some cases for wide differences of opinion on certain questions. In order to summarize briefly, differences in manuals will be pointed out only if the data make it necessary.

1. Only forty per cent of the teachers felt that the introduction to the manual they used added to their understanding of the teaching of arithmetic. The teachers stated that it was helpful when the material dealt with place value and the development of arithmetic meaning. Those that disapproved either had not bothered to read it and so stated, or complained that the same material is found in professional books.

2. Seventy per cent felt that unnecessary space was devoted to the chapter overviews since the material given in the overview was already available in the page suggestions.

3. Only fifty-five per cent used the suggested procedures even though ninety per cent agreed that the procedures were easy to follow. In general, they deviated primarily when the suggested procedures followed the text page or when the suggestions required the use of concrete or semi-concrete materials.

4. Only twenty-five per cent used the concrete or semi-concrete materials suggested. The comments indicated that the teachers do not use these materials either because they do not have them available and feel there is too much work involved in their preparation or because they think they are too consuming of class time or unnecessary to the understanding of the lesson. Another possible reason for this situation exists in that only fifty-five per cent felt that the authors gave a good explanation of how the suggested concrete materials should be used. The last per cent quoted may be misleading in that this was one question where the manual used made a great difference. Those using one of these manuals were almost unanimous in stating that their manual did a good job while those using the other manual said just the reverse. It is significant, too, that teachers with more than three years of experience used two or three times as many of the suggested concrete materials as did those with less experience.

5. Only twenty-five per cent of the teachers consistently used the enrichment activities. In this case, the reasons for lack of use depended upon the manual being used. An analysis of the teachers comments indicated that the major objection on the part of the people using one of these manuals stemmed from the amount of time, supervision, and teacher explanation necessary for the satisfactory use of these exercises. The people using the other manual objected because they felt that the activities given were not interesting, challenging, usable, or practical.

6. None of the relatively inexperienced teachers and only a few of those with more than three years of experience used the films, filmstrips, professional books, pamphlets, and free materials listed for their convenience in the back of their manual. The one teacher who had sent for free materials said that he had been disappointed by the amount of useless material he had received.

It is noteworthy that the teachers in answering the questionnaire showed that at least the one lesson observed was typical of their everyday performance in that what they did the day of visitation corresponded closely with what they said they usually did.

Features the Teachers Liked

1. The teachers were unanimous in their approval of the format of the teacher's edition that combines the pupil text and teacher's manual under a hard-back cover. If the manual suggestions for a text page and the text page are not both on the same page in the teacher's edition, they would definitely like them to be.

2. Seventy-five per cent agreed with the authors' philosophy. From the comments made by the teachers it is doubtful that they were clear as to what was meant by "philosophy." The reasons given for disapproval showed that there was not a fundamental difference in philosophy but rather a disagreement based on a few specific methods recommended in the manual.

3. Seventy per cent felt that helpful suggestions were given for improving pupils' problem solving ability. Their comments indicated that they realize this is a difficult problem, but they felt that only the usual things were suggested.

4. Seventy per cent followed the testing program outlined in the manual. Their comments indicated that at times they deviated by adding problems and examples from other sources and that they would like more items that really test for understanding rather than just computation.

5. Seventy per cent of the teachers who had pre-book lessons provided in their manual approved of this feature. They indicated that they approved because this type of lesson created interest and was a good way to approach a new process since the child was allowed to think for himself.

6. Eighty per cent found the manual helpful in planning lessons. Their comments showed that it was helpful because many suggestions were given and new ideas were brought to mind. They disapproved when the page suggestions given were general, not practical, or procedures followed the text page.

The Task of the Manual Authors

An examination of a number of modern arithmetic manuals inevitably leads to the conclusion that the authors have undertaken the presentation of an extensive program in arithmetic—the development of computational skill and problem-solving ability, provision for individual differences through ability grouping and/or enrichment activities, testing and diagnosis, mathematical background for the teacher, and teaching suggestions for each text-page—in accordance with their philosophy and value judgments. Certainly the task is a difficult one. It is inordinately complex since professional integrity in the form of adherence to sound theory and strict mathematical correctness must be maintained while attempts are made to meet the needs of arithmetic teachers who differ greatly in ability, experience, and preparation.

Another, and I believe, even greater complication arises from the textbook series

itself. Regardless of the statement of authors of arithmetic series to the effect that the text cannot teach, the textbook is still written in a style that makes this a definite possibility, even highly probable. The basic weakness of the manual lies in the fact that it is, so to speak, a witness after the fact. In other words, after the textbook has been written, the authors begin work on the manual. This results in the page suggestions being tailored to fit what is already in the text. Thus in many cases, and many teachers are becoming aware of this fact, the teacher is confronted with a situation in which all the authors claim to be teaching arithmetic meaningfully but some of them are actually encouraging the opposite effect by presenting not only a text that does the "telling" and "showing" but a manual which encourages the textbook approach.

Although there are undoubtedly many teachers who are teaching arithmetic as they were taught twenty or more years ago, there are also many well-qualified teachers who look to the manual only for the newer knowledge that has become available concerning the teaching and learning of arithmetic. The manual authors have a responsibility here in that the manual may well act as a bridge to close the gap between theory and practice. Page suggestions which merely tell the teachers to follow the text-page are quite worthless since teachers who would follow such suggestions have probably been doing just that.

If as Thiele says:[3]

> Psychologically speaking, not even the teacher can give children meanings. They must be discovered by children themselves. Teachers can do no more than guide, stimulate, create situations, bring about conditions, and, generally speaking, help children gain meaning. If the principle of discovery be sound, there will be small need for "demonstrating" or "explaining" or "telling" or "showing" on the part of teachers.

many authors are merely paying lip service to this philosophical strand—the meaning theory. It seems patent that children cannot discover if they are never given an opportunity to do so. When the text or teacher explains the process and gives the rule so that processes can be quickly mastered, the children may or may not understand the relationships. While recognizing the fact that not all authorities would agree, I believe that this mode of operation postulates that understanding comes after mastery not before it. The use of the pre-book lesson which has been so well received by teachers would seem a logical solution to the discovery approach-textbook method dilemma. With full utilization of this type of introductory lesson in a textbook and manual planned simultaneously so that every advantage could be taken of the knowledge available regarding optimum learning conditions, arithmetic instruction could be immeasurably improved.

The recognition of the aims of arithmetic as being both mathematical and social has led to increased emphasis on the teaching of relationships within the number system and the use of social settings. The value of the former is obvious but the application of the latter has led to extremes by some authors. Certainly teachers have a right to expect practical suggestions rather than such an extreme as presented by one manual which proposes the use of available community resources to the point where it seems logical to conclude that the social studies might be dropped from the curriculum and taught as an adjunct to arithmetic. The use of business forms, graphs, charts, tables, diagrams and the like have a legitimate place in arithmetic instruction but the undertaking of a unit in communication, as suggested, to add meaning to a page of problems centered about finding the cost of telegrams seems difficult to justify. It also seems hard to conceive that a teacher would take a class on a field trip to a dairy in order for pupils to secure prices that might then be used in making up original problems. Certainly such a field trip would be taken for other major purposes and the information secured in respect to prices might then be used for such a project but as a by-product not as a major objective.

[3] NATIONAL SOCIETY FOR THE STUDY OF EDUCATION. *Fiftieth Yearbook. Part II.* "The Teaching of Arithmetic." Chicago: University of Chicago Press, 1951, p. 80.

Since the modern philosophy of education places emphasis on caring for individual differences, it is inevitable that the authors of arithmetic manuals must face this issue. All authors do discuss the subject in their introductory chapter and the application of their program, if they offer one, is found in the suggestions given for the text-page under the label of either enrichment or differentiations. Here again, the teacher has a right to expect some down to earth advice. Certainly a teacher does not need to be told to help pupils who need more practice or that superior children can be used as assistants or be allowed to read for fun or engage in some other special interest activity that will be profitable. If ability grouping is advocated, more is needed than the admonition to adjust methods, demonstrations, and explanations in accordance with the needs of the groups.

The writer violently disagrees with the authors who believe that the arithmetic period in the case of the superior child can better be spent on science or social studies or be devoted to art, music, dramatics, or some other form of creative work. With an understanding of mathematics becoming increasingly important in modern living, it seems that the time provided for arithmetic could very profitably be spent on arithmetic with the superior student being given the opportunity to engage in activities which will enrich his appreciation and understanding of our number system. Many of the activities provided in manuals are not of this type and could be classified under the term "forced enrichment." By this term, I mean to imply that the authors have taken the text-page and with determination have written something which they call enrichment activities even if the material has little or nothing to add to the understanding of arithmetic. The extensive use of the social studies as a main source of enrichment material falls in this category and is both impractical and unsound. Since it is probably true that the majority of our teachers are ill-equipped for the task of enriching the background of the superior student, it is imperative that the manual present activities that are mathematically worthwhile.

There can be no quarrel with the use of manipulative material when necessary although it does seem that their proper use in the primary grades would eliminate the necessity of continuing this process at the sixth grade level except in the case of those who have failed to acquire simple, basic understandings in the earlier grades. If such materials are considered essential in the teaching of a particular page, it seems obvious that the authors should give an adequate explanation of their use. A favorite procedure of some authors is the suggestion to make an enlarged drawing of an illustration in the book. No explanation is given as to why this is necessary or how the teacher should use it. The statement in itself is ambiguous. Do the authors mean an enlarged drawing on the blackboard which would be fairly simple to execute or do they mean an enlarged drawing on oak-tag which would be a time consuming project indeed? What useful function is served by such a drawing assuming that each child has a copy of the text and therefore, a picture of the illustration before him? It is difficult to believe that teachers would follow such a suggestion or they have indicated that they do not. If authors consider these materials essential I have concluded that they will have to do a better job in selling them to teachers. It might help if notations were made as to where these articles may be purchased since teachers show very little inclination to make them.

In view of the teacher shortage and the little preparation in arithmetic required by some institutions, the inclusion of a considerable amount of mathematical background for the teacher can certainly be regarded as a legitimate function of arithmetic manuals. The teachers have indicated that they are interested in material of this type if it is included in the page suggestions. Judging from experience in visiting many different schools and observing arithmetic classes, it seems likely that the authors who include this type of information are doing a real service. It cannot even be assumed that all teachers use the terms *remainder* and *differ-*

ence, *fact* and *combination*, or *example* and *problem* correctly.

The combined text-manual with the answers on the text page rides the peak of popularity and surely will be accepted as standard. The manual is of no value if the teacher does not read it and this format makes it highly unlikely that he will be able to escape doing so. Of course, this only adds to the authors' difficulties, since the indications are that they will have to present all essential material within the limits allowed by the text page margin. Teachers have indicated that they do not read the introductory chapter or overviews with any serious intent.

The manual is in a unique position to help orient teachers to the latest theory. If the material is not in line with sound theory, well-planned, and presented in a practical fashion which makes sense to teachers, it is not fulfilling its proper function. Not all manuals meet these specifications. If discovery, as stated or implied in manuals, is to be the basis of aiding children in understanding arithmetic, a great deal of revising of manuals and textbooks will be a necessary corollary.

EDITOR'S NOTE. Dr. Folsom has made a study of Teachers' Manuals in arithmetic and has sampled twenty-two classrooms to determine how teachers use the manuals which are provided. With this background she not only has presented the results of her survey but also has some good advice for authors.

The purpose of a manual is to enhance the instructional program. Can a manual in the usual limitations of space do all the different things that a wide variety of teachers really need? Some teachers need background in the mathematics of their subject. Some need advice on the methods of presentation. And still others may need help with the organization of a group of thirty pupils for optimum learning by each. If a manual attempts to do all of these and also includes such other bits of advice as authors think they should have then a professional book for teachers is being offered and intelligent teachers with good backgrounds will not want to be bothered. But "who will bell the cat?" i.e. who will see that the teacher who really needs the guidance of a good manual will be the one who uses it? The problem of the manual is something like trying to fit all women into a size fourteen dress.

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Mathematical Ability and Masculinity

PHILIP LAMBERT
University of California, Los Angeles[1]

THE IDEA THAT MEN ARE BETTER than women at mathematics has been widespread. Many writers on education have taken it for granted; a few have tried to prove it. Unfortunately, the general acceptance of this assumption distorts the test results; achievement rather than innate ability is measured. A comparison of the scores of men and women on various kinds of arithmetic tests is not conclusive, since ability to do arithmetic is partly the result of past and present interest in the subject. Even at an early age, boys are expected to be interested in mathematics. Girls, on the other hand, though they may have equal ability, may be discouraged from learning by the prevailing idea that mathematics is a masculine field.

A number of studies have been made comparing the arithmetical ability of boys with that of girls. In most, but not all, cases the boys have been found to be superior (2, 3, 4, 7, 9, 12, 13, 16). Similar results have been obtained with adults (1, 5, 8).

Based on these and similar studies, psychoanalytically oriented writers have said that arithmetical ability is a masculine trait, and that high arithmetical ability on the part of females implies a masculine orientation. In a recent article, Plank and Plank (11) investigated emotional components in the learning of arithmetic through a study of childhood experiences as revealed in the autobiographies of 13 men and 3 women. In the case of both men and women who later showed unusual interest in mathematics or became mathematicians, they found that the early relationship with the mother was weak or broken. Girls who grew up to be mathematicians also showed a "denial of femininity" and "a strong identification with a male figure."

The purpose of the present study was to test these observations directly and experimentally by investigating the relation between arithmetical ability and masculinity or femininity. The following hypothesis was posed: that women who have high interest in and ability for mathematics are more masculine than women who have only average mathematical ability.

Procedure

Subjects. The 1372 *Ss* were undergraduate students at the University of California at Los Angeles. Group I consisted of 80 students enrolled in advanced courses in mathematics or physics, most of whom were majoring in these subjects. Group II contained 1292 senior students entering practice teaching over a three-year period.

The mean number of college units in math taken by the *Ss* in Group I was 19.5 for the men, and 26.8 for the women. For Group II, the mean was 4.6 for the men, and 2.3 for the women.

All *Ss* were given two arithmetic tests: 1) Section A of the arithmetic examination of the American Council on Education, and 2) a Basic Arithmetic Skills Survey test created by Dr. John Gowen. These tests are among those given to all senior students entering practice teaching at the University. Scores of the two tests were combined to produce a single arithmetic proficiency score for each subject.

In addition, all *Ss* were given the Minnesota Multiphasic Personality Inventory (MMPI). Scores on the arithmetic tests described above for all *Ss*, and scores on the Mf-scale* for Group II, were obtained from college records. To get an Mf-score and a combined arithmetic score for Group I, the *Ss* were given separate tests consisting of the

[1] Now at the University of Wisconsin.

* Masculinity-femininity scale.

TABLE 1

ARITHMETIC TEST—INDIVIDUAL *t*-TESTS

	N	Respective Means	*t*	Confidence Level Reached
Group I women×Group I men	10 70	32.10 *and* 29.39	1.28	$P<.17$
Group I women×Group II women	10 744	32.10 *and* 18.39	Not needed	$P<.01$
Group I men×Group II men	70 448	29.29 *and* 21.45	9.87	$P<.01$

TABLE 2

Mf-SCALE—INDIVIDUAL *t*-TESTS

	Group I		Group II		*t*	Confidence Level Reached
	No.	Mean	No.	Mean		
Men	70	29.39	448	26.70	3.58	$P<.05$
Women	10	28.40	744	37.60	6.91	$P<.01$

TABLE 3

CORRELATION BETWEEN ARITHMETIC TEST AND Mf SCALE (MMPI)

	Group I	Group II
Men	+.008	−.025
Women	+.156	+.005

total MMPI scale and the two arithmetic tests mentioned above. (The *S*s were not informed of the purpose of this test.)

Results

Mean scores on the arithmetic test are given in Table 1. Female math majors scored 32.1, while male math majors scored 29.39. Application of a "t-test" revealed that there was no significant difference between these scores. Female non-math majors had a mean score of 18.39, and male non-math majors had a mean score of 21.45. As might have been expected, there was a significant difference between the arithmetic test scores of female math majors and female non-math majors, and between male math majors, and male non-math majors ($P<.01$).

Mean scores on the Mf-scale of the MMPI for all groups are shown in Table 2. Male math majors scored 29.30, while male non-math majors scored 26.70. Application of a "t-test" showed that this difference was not statistically significant ($P<.05$). There was, however, a significant difference in the case of the women. Group I women (math majors) had a mean score of 28.40, while Group II women (non-math majors) had a mean score of 37.60. That is, math majors were not only equal to non-math majors in femininity, but significantly more feminine.

Since the *N* of women in Group I was so small, it was decided to check these results by taking a new sample. Accordingly, the MMPI was given to 10 additional female math majors, selected at random from college files. The average number of units of math taken by these subjects was 32.1. The mean Mf-score of this group was even lower: 26.30.

There was no significant difference between the mean Mf-scores of this second sample and the first (t-ratio=1.39). However, there remained a significant difference between the second group of 10 female math majors and the 744 female non-math majors. We can therefore conclude that there was in fact a high significant difference in femininity of interest pattern between female math majors and female non-math majors.

There was no correlation found between arithmetic proficiency and masculinity of interest pattern for either Group I or Group II. (See Table 3.)

A Bartlett's Test for Homogeneity of Variance gave an $x^2=12.94$ with 3df. This is significant at the .01 level of confidence. In order to run a simple analysis of variance it is desirable to have an insignificant x^2. Since the "F" ratio was significant at the .001 level and since *all* the *significant* "t" values were well above the .0001 level it is justifiable to overlook the results of the Bartlett's Test.

Discussion

The Mf scale measures masculinity or femininity of interest pattern in terms of current cultural stereotypes. The results of the present study suggest that there is no relation between mathematical ability and unusual masculinity of interests in the case of either men or women. A possible conclusion might be that the three women mathematicians whose autobiographies were studied by Plank and Plank were atypical. These three women, however, lived in nineteenth-century Europe, where masculine and feminine interest patterns were more divergent than in present-day Minnesota. By the standards of their society, they might have been considered outstandingly masculine. Plank and Plank's finding that men and women who became mathematicians were apt to have a strong emotional tie to their fathers, and a weaker or broken one to their mothers, has yet to be investigated.

If Plank and Plank's findings are accepted, one possible hypothesis which might be posed as a result of this study is that the fathers of girls who become mathematicians are psychologically feminine. Another possibility would be that the female math major, simply because she has a strong positive relation with her father (rather than a weak or hostile one) is or becomes more feminine.

Although girls who are math majors were found to be more feminine, there was no correlation found between masculinity or femininity and arithmetic proficiency. (Among non-math majors, it appears that the girls as a whole take fewer math courses and do less well on arithmetic tests; thus conforming to the cultural stereotype that women are less interested in mathematics and have less ability in this field.) One might therefore postulate that among girls who are good at arithmetic, it is actually the more feminine ones who become mathematicians.

Summary

Various writers have suggested that mathematical ability is a masculine trait, and that women who become mathematicians have a high masculine orientation. This study was designed to test that hypothesis. Male and female college students, some of whom were mathematics majors, were given an arithmetic test and the Mf-scale of the MMPI. There was no significant difference found between the mean scores of the male math majors and male non-math majors on the Mf-scale There was a statistically significant difference in the mean scores of female math majors and female non-math majors; however, contrary to expectations, math majors were not only equal to non-math majors in femininity, but more feminine. There was no correlation found between arithmetic proficiency and masculinity of interest pattern for either sex.

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Arithmetic in the School Camp

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SCHOOL CAMPING IN THE UNITED STATES is an exploding idea. Over five hundred school systems representing twenty-four states have school camping programs. These programs vary, but there is a general trend toward a one week program for sixth graders. In the type of program this article considers, pupils and teacher together assume direct responsibility for designing, carrying out, and evaluating their plans. These activities are selected by the groups on the basis of being the best way to reach certain educational goals which can be attained better by group living in the outdoors than in a classroom.

This extension of the classroom is designed to provide an environment for the development of concepts through direct experience, and for the application of skills learned in school. Arithmetic, science, language arts, social studies and fine arts are all centered around a core problem. Living together around the clock, pupils and teachers use the entire range of talent found among themselves to solve the real problems of people in communities: food, shelter, clothing, industry, recreation, and wise use of resources for the best general welfare. In solving these problems, each individual can find inspiration, adventure, responsibility, and a new sense of freedom. For the classroom teacher, this is an exciting new classroom, a room with moveable walls which can be pushed out to embrace the whole countryside.

Through camp activities, certain specific objectives of arithmetic instruction can be attained more effectively than might be the case in the classroom. For example, the learner has an opportunity to use his understanding of quantitative relationships in a natural environment. He can use knowledges and skills learned in the classroom to meet social needs. He can define and analyze problems in order to determine the information needed to solve them. Through his own activities he can understand the primitive devices that were invented by men to keep accounts and tell time. He can develop a spirit of inquiry and respect for the orderliness of his natural environment. Furthermore, he can find an opportunity to use the abstractions of a quantitative vocabulary with meaning derived from direct experience. In addition, he can demonstrate quantitative concepts through practical application.

It is with this last objective, the development of numerical concepts through school camp experiences, that this paper is concerned. Arithmetic concepts are relative and perceptual. Whatever we conceptualize, we perceive as relative to other concepts derived from our unique previous experiences. That is, we see only what we have experience to see. Since our perceptions come not from objects but from us, our ideas about any object depend upon our point of view. Then we act according to our perception of the immediate situation in which we find ourselves. Concepts, then, come from our personal selection and organization of perceptions. Therefore, no one actually sees a thing the same as others see it or has concepts identical to those of anyone else. In order to communicate these concepts, we use abstract vocabulary, therefore, the accuracy of communication is relative to the accuracy with which we have perceived and organized our ideas about quantity, space, and time.

This relationship between concepts and experience is demonstrable in school camping. Children ask, "How far is it to camp?"

The abstractly-expressed or measured distance from one city to its school camp is fifteen miles. This distance was a two-day wagon trip for the pioneers with a load of grist, because they had to make a roundabout river crossing. According to one contemporary farmer, the distance is "One good cigar." In fact, the trip takes about forty-five minutes by school bus. One school camper anticipates that distance as the measure of his first prolonged separation from his parents; another thinks of it merely as a field trip to the lake where he fishes with his father. Within the camp, the quarter-mile hike to Mystery Valley is a quick walk, but on a hot summer day when the underbrush is leafy, the same hike becomes a jungle safari. In such instances, school camping affords opportunities for pupils to refine and accurately express their concepts of distance.

Three Kinds of Questions

Children who go to camp ask questions in regard to three kinds of arithmetical concepts. These may be grouped as questions about quantity, questions about space and distance, and questions about time.

The following questions about quantity represent those that children ask while they plan and carry out activities:

How much will it cost to go to camp?
How much clothing shall we bring?
How many people can sleep in the bunkhouses?
How many tables will we need to set for lunch?
How many fires will we need for our cookout?
If each person uses one-fourth pound of hamburger, how much shall we bring?
How many packages of buns do we need?

The next set of questions relates to the campers' concepts about space and distance:

How large is the bunkhouse?
How big is the dining room?
What is the circumference of the tree trunk?
How steep is the slope?
How would a scale profile of the oil well look?
What scale shall we use for our map?
How far is it to camp?
How far is it across the lake?
How high is the water tower?
What is the shortest way to the Grist Mill?
How deep is the top-soil?
How deep is the gravel-pit?
How do I use a compass?

When children seek answers to the following questions, they are forming and revising their concepts about time:

How long will we be in camp?
How long until lunch?
When should we build the fire?
How old is that tree?
What achievements have people made since the tree began to grow?
How much longer will it take to walk than to ride to the lumber mill?
How long ago was the house destroyed at the Abandoned Farm?
How long does it take an inch of topsoil to form?
When did the light we see leave that star?

Many similar questions which involve quantitative, spacial and time concepts arise in varied contexts. They occur while the children are planning the trip to camp, selecting and planning activities, hiking, boating, repairing a bridge, making a terrarium, gazing at the stars, adventuring with compasses and maps, making ice cream, dying cloth with native dyes, exploring the lake shores, cutting ice, skiing, visiting a rural village, lumbering, exploring a mineral deposit, reconstructing the history of an abandoned farm, or cooking over a camp fire.

Examination of one question will show some ways in which a teacher can use it to guide the development and reorganization of concepts to approach reality. In so doing, the children will discover relationships through first-hand observation and activity. For example, sixth grade children frequently ask "How far is it across the lake?"

Because perception comes from the individual, the teacher must try to understand the child's motive for asking. To do this, the teacher must make a judgment as to whether the questioner is seeking information, playing a social role, or hypothesizing. According to the leader's perception of the learner's reason for asking the question, the leader acts on one of three alternatives. He may (1) ignore the question, (2) answer the question, or (3) test the child's reason for asking the question. If he decides that the question is important to the children and in line with educational objectives, he will help the children discover ways of measur-

ing the distance. The question becomes a situation for learning ways of estimating and measuring distance.

Development from a Question

Such a situation may take the form of a conversation between a camp teacher and a group of children standing on the beach. The conversation may proceed something like that which follows:

CAMPER 1: How far away is that point across the lake?
TEACHER: I don't know. How far do you think it is?
CAMPER 1: A half mile?
CAMPER 2: No, a half mile is farther than that!
TEACHER: Do you see anything that looks like it is a mile away?
CAMPER 2: Yes, that hill at the end of the lake.
TEACHER: How did you get your idea of how far a mile is?
CAMPER 2: I ride with my father a lot. I look at the speedometer and then pick a tree I think is a mile off. Then I check it.
CAMPER 3: I know it is a half mile from my house to school. It isn't that far across the lake.
CAMPER 1: Well, how far *is* it across the lake?
TEACHER: Maybe we can find out.
CAMPER 4: How?
CAMPER 2: We can row across with a line and then measure the amount of line we use.
CAMPER 3: That tree across the lake looks about as far as that tree down the shore. We can measure the distance to the tree down the shore.
CAMPER 1: You can get a yardstick. That takes too long for me.
TEACHER: Maybe there is a faster way.
CAMPER 2: I could walk there and count my steps.
CAMPER 5: My legs are longer. I could get there in less steps. How will you know how far it is when you know how many steps you walk?
CAMPER 2: I can measure my steps.
TEACHER: How?
CAMPER 2: At school we figured out our room was thirty feet long. It took me fifteen steps to walk the length of it.
TEACHER: Then how long is each of your steps?
CAMPER 2: Two feet.
TEACHER: Then let's see how many steps you take to get down to that tree on the shore.
CAMPER 5: I know a way our Scout leader taught us.
TEACHER: How does it work?
CAMPER 5: I can hold a stick at an arm's length and see how much of it blocks out that tree across the lake from my sight. I can measure the stick and measure my arm. It takes two inches of the stick to cover the tree. The stick is at right angles to my arm and two feet from my eye. How high is that tree?
TEACHER: Why do you want to know?
CAMPER 5: Well, if I knew how high the tree was, I could figure that since my arm is twelve times as long as the stick, the lake would be twelve times as wide as the the tree is high.
TEACHER: The tree is about sixty feet tall.
CAMPER 5: Then the lake is twelve times sixty or seven hundred twenty feet across.
TEACHER: How many yards is that?
CAMPER 5: Seven hundred twenty feet divided by three is two hundred forty. The lake is two hundred forty yards wide.
CAMPER 2: It took me three hundred forty-five steps to that tree.
TEACHER: How many feet is that?
CAMPER 2: One step is two feet. Three hundred forty-five multiplied by two is six hundred ninety feet.
CAMPER 3: That is about two hundred thirty yards.
TEACHER: How much difference is there in the answers?
CAMPER 1: Ten yards, and that is pretty close.
TEACHER: How else can we measure a distance when we can't go directly from one point to another?
CAMPER 6: I figured two hundred yards.
TEACHER: How did you estimate the distance?
CAMPER 6: My casting line is fifty yards long. I figured it would be four casts across the lake if I let all the line out.
CAMPER 7: I know another way. It took me ten minutes to row over this morning. It takes me twenty minutes to row a quarter of a mile down our lake at home to where my buddy lives. If it took me half as long to cross this lake as it does to row to my friend's cottage, it is half as far across. One-half of one-fourth mile is one-eighth of a mile. 5,260 feet is one mile, so across the lake is 675½ feet.
TEACHER: Could anyone use a compass to find out how far it is without actually crossing the lake?
CAMPER 8: You could find out the exact direction from this point to the tree across the lake. There are 360 degrees in a circle. The tree is at zero. It is straight north! Then we can hike west, or at 270 degrees, to the end of the lake, go north, 360 or zero until the tree is straight east, (ninety degrees), and come back to the tree. If we pace the distance we have to go north we will have a line overland just as long as across the lake. Here, I'll draw it on the ground.
CAMPER 9: There is another way in which we won't have to walk so far. I read about it in a book. It is called triangulation. Two sides of a right triangle squared are equal to the square of the hypotenuse. Therefore, we can use a compass to go down the lake one direction until we can sight overland at a right

angle to the tree. Then we'll turn and go to the tree. We will pace off each direction. Then we have to square each distance and add them together. The square root of the sum will be the distance across the lake.

TEACHER: Will your answer be expressed in steps, feet or yards?

CAMPER 9: Oh, I forgot to change paces to yards. I'll draw a diagram in the sand to show you how it is shorter than the other way where we'd have to go around three sides of a rectangle.

TEACHER: Shall we try it? What else must we learn?

CAMPER 7: How to get right angles with a compass.

TEACHER: You suggested several ways to estimate or find out the distance. Let's take a few minutes to recall them so our recorder can write them down for the camp journal. Which way was easiest? Which way most accurate? Which way do you think it took people longest to discover? Can we use these ways to measure other distances?

Such a discussion leads to taking a compass hike and plotting a scale map of the lake. In addition, a follow-up unit at school or an individual project may be undertaken to investigate the ways man measures distances and the ways in which these methods were developed.

In the group discussion cited above, individuals were thinking at different conceptual levels, building on the discoveries of one another, drawing from previous knowledge and experience, and using their peer audience to test their ideas. To help the children build concepts, the teacher used the open space, the long period, the high motivation and the problem situation which were provided by the camp. In seeking the answer to their own question, "How far is it across the lake?" the children were developing relative concepts of miles, feet and yards; direction, shape and position; various ways to estimate; and the intellectual development of our ways to measure distance. The example demonstrates how relationships were perceptual and how perceptions differed relative to the differences in the past experiences of the individuals.

EDITOR'S NOTE. School camps and the accompanying "outdoor education" have been increasing in recent years. Teachers in these situations do have opportunities in environmental setting and in arithmetical topics which can be explored more informally than in the typical classroom. Ideally, we should do things at the camp that are suited to the circumstance and reserve for the classroom those things which can be done better in that situation. No longer are the camp enthusiasts claiming that anything done in a schoolroom can be done better in a camp situation. As in all learning situations, very much depends upon the leader. The good teacher in a classroom may not be the good one in a camp. The techniques for guiding learning seem to differ. The alert camp leader will recognize good learning situations and will capitalize upon them as illustrated by the questions and answers given by Miss Roossinck.

Algebra in Grade Five

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I SHOULD LIKE to make known to arithmetic teachers an experiment that I have been and am still conducting. Experiment is perhaps the wrong word. In Europe, as a matter of course, the abler students begin algebra at 10 or 11 years of age. American students are just as intelligent, and I never had any doubt that they would be able to do the same. So perhaps it should be called a demonstration rather than an experiment.

Shortly before Christmas 1958, Mr. Martin, principal of Middletown Central School, kindly allowed me to teach a class for one period each week. His school used homogeneous grouping; there were three sections of Grade Five; I taught the quickest of the three. There were 26 students in my class.

There was no doubt that the students were ready for algebra. In fact, my main problem was to keep the abler ones busy. I was not using any textbook. No American text exists for teaching algebra to Grade Five. The English texts are designed for students of this age, but there are references to pounds, shillings and pence, cricket etc. So I contented myself with explaining some idea and setting a dozen problems on it. Some students might require individual attention, but I found it hard to give them this, because the best dozen or so students would finish the problems very quickly and be demanding something more to do.

This year I am continuing with these same students, who are now of course in Grade Six. Again, they are having one lesson a week in algebra. Last year they absorbed the basic idea of what algebra was, what x meant and how it was used to express general truths about arithmetic. This year we are working through an English text, *A School Algebra* by H. S. Hall. These cost less than $2 each, and contain enough information to keep the students busy for several years. With a mixture of oral and written work, we get through twenty or thirty problems in each period.

It is of course essential to approach algebra on the basis of understanding and genuine discovery by the students. Last year I used much of the material given in *The Mathematics Student Journal*. The leading articles, November 1958 through May 1959, were devoted to the transition from arithmetic to algebra.*

Some points that might be stressed by arithmetic teachers with the object of making it easier for students to learn algebra are the following.

1. Continually ask students how they picture arithmetical expressions. For example, how do you picture $3+2$? Answer, 3 objects and two objects put together, 0 0 0 I I. How do you picture $3-2$? Three objects, then two removed. 0 Ø Ø. How do you picture 3×2? Three rows of two objects, or two rows of three objects,

0 0 0
0 0 0.

And so on. In algebra, the important thing is to see clearly *what you are doing;* the answer is important, but less so.

2. Emphasize that $2\times3+4$ means 10 and not 14. In algebra one has expressions like $2n+4$ corresponding to "Think of a number, double it and add 4." This is *not* the same as "Think of a number, add 4, and double the result." To get the latter one has to use $2(n+4)$; in the arithmetical case, where someone has thought of the number 3, one would need to write $2\times(3+4)$.

3. Arising out of the last point, it would help greatly if students became accustomed to the meaning of parentheses in arithmetic.

4. In algebra, one often needs a repeated product, such as $2\times3\times4$. These do not seem to occur in commercial arithmetic. They occur perhaps in finding volumes of rectangular blocks. It would be helpful if the meaning of such expressions could be stressed.

I do not know if I shall be able to produce any statistical evidence by the end of this year. My impression is that students enjoy this algebra period and look forward to it.

The teaching of algebra has not been part of the duties of a Grade Five teacher in the past. The difficulty in introducing algebra earlier is clearly that of finding teachers who can present it effectively rather than of finding students who can absorb it.

I should very much like to be in touch with any college training elementary school

* The first article, *Why Is Arithmetic Not the End?* was reproduced in THE ARITHMETIC TEACHER, March, 1959.

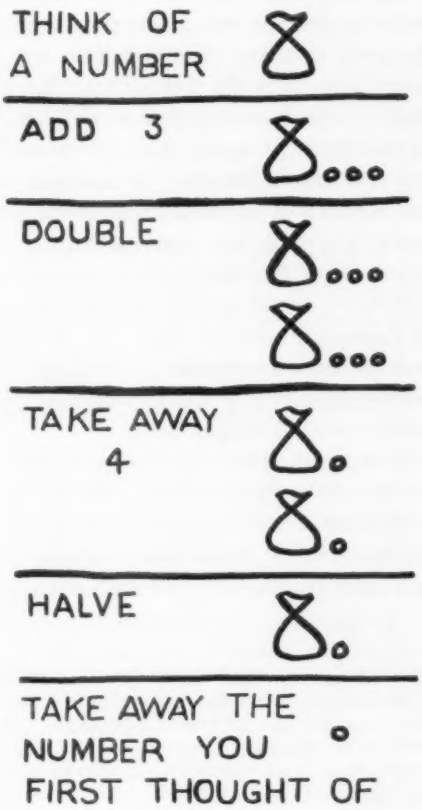
teachers, or with any education authority, interested in making changes of this kind. It clearly cannot happen all at once. But if we can teach algebra to some of the abler Fifth Graders now, some of these will become teachers when they grow up, and they will think it quite natural to teach this subject at that stage.

Algebra in "Tricks"

The knowledge we get from these tricks is just what we need when we begin to apply mathematics to science and engineering.

Probably you have often heard tricks like this one:

Think of a number.
Add three to it.
Double.
Take away 4.
Halve.
Take away the number you first thought of.



Whatever number you think of, provided you do not make any mistakes, you will end up with the answer 1.

Why does this work? We can see by thinking in pictures. When you have thought of your number, we will suppose you put that number of stones in a bag. I can see the bag, but I do not know how many stones are inside it.

When I say, "Add three," you put three more stones next to the bag. I can now a see bag and three stones. I tell you, "Double." You bring up exactly the same things again. I now see two bags and six stones. "Take away 4." You remove 4 stones. I see two bags and two stones. "Halve." You do so. I now see one bag and one stone. "Take away the number you first thought of." The number you first thought of was the number of stones in the bag. So you remove the bag. That leaves just one stone. So the final answer is always 1—regardless of the number you may have chosen.

EDITOR'S NOTE. Do not be alarmed with a proposal to teach algebra in grade five. If algebra involves thinking with and about an unknown quantity and if it includes establishing generalizations, then many teachers are beginning algebra in grades one and two. $3+?=9$ is essentially the same as $3+N=9$ and both of these are equations. In later grades when pupils discover that the area of a rectangle may be found by multiplying the number of units of length by the number of similar units of width and when this is expressed in a formula as $A=LW$, this certainly becomes an algebraic pattern. Likewise work with fractions may be algebraic as for example the simple equivalence

$$\frac{3}{4}=\frac{?}{24}.$$

The "algebraness" of arithmetic depends in large measure upon how children are led to think about things they do in arithmetic. If they learn mechanically without thinking, this is neither good arithmetic nor algebra. Twenty-five years ago several states and cities began to speak of "Mathematics for Elementary Schools" in order to encompass the elements of algebra and geometry that one finds in a good elementary school program. Many teachers will recall the sterility of the older memory-drill programs in arithmetic and the very narrow view of arithmetic that was current thirty years ago.

People who are interested in the experimental work which Dr. Sawyer is doing in Connecticut may write to him at Wesleyan University.

Retention of the Skill of Division of Fractions

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ONE OF THE QUESTIONS constantly confronting teachers is that of retention of any skill which is taught. In most cases there is no direct check on retention of any particular skill either on a basis of the group of children as they move along through the grades, or on a basis of the individual students. This particular study was a follow-up of a previous study in which the learning of the skill of division of fractions had been the main concern; attempting to determine the relative degrees of retention provided interesting and valuable supplementary information.

In the previous study mentioned above,[1] three groups of sixth grade students were designated as one experimental and two control groups. The experimental group was taught by the common denominator, method, and both control groups were taught this skill by the inversion method. The evidence gathered in this study indicated that there was no significant difference between the skill attained by the students as a result of either method of teaching. However, one question of interest which could not be answered immediately was that of the relative retention of this skill by the students who had learned it by the different methods. The follow-up study was concerned, then, with answering this question: *Is there a significant difference between the retention of the skill of division of fractions by the children taught by either the common denominator or the inversion method?*

[1] Lois Evans Stephens, "A Comparative Study of Procedures Used in Teaching Division of Fractions in Sixth Grade Arithmetic," (Unpublished Master's thesis, University of California, Los Angeles, 1957).

Limitation of the Problem

This study was limited to an attempt to answer the above question in relation to the students involved. The students included in this study were those who had been a part of either the experimental group or one control group in the previous study.

Related Research

A thorough search of recent literature revealed no report of a study similar to this one or its forerunner; however, there were references to other studies which were of general interest in the field of teaching fractions. In the *Review of Educational Research*[2] the statement is made that much controversy remains in methods of teaching fractions. Reports of various studies in the teaching of fractions are summarized in the *Encyclopedia of Educational Research* as follows: (1) Studies reveal a large amount of error and confusion with fractions. (2) Teaching fractions beyond common adult usage is not recommended. (3) Textbooks develop fractions far beyond usage.[3]

Among individual studies, the correlation between understanding of the meanings and computational skill was investigated by Rappaport.[4] It is his recommendation that much more emphasis be put on the teaching of understanding of meanings.

[2] E. Glenadine Gibb and H. Van Engen, "Mathematics in the Elementary Grades," *Review of Educational Research*, 27 (1947), 329–342.

[3] Walter S. Monroe, *Encyclopedia of Educational Research* (New York: The Macmillan Company, 1950), pp. 44–58.

[4] David Rappaport, "Understanding Meanings in Arithmetic," THE ARITHMETIC TEACHER, 5 (1958), pp. 96–99.

Three other individual studies were concerned with increasing understanding of the process of division of fractions. One sets forth a rationalization of the process of inverting the divisor and multiplying through the use of reciprocals.[5] Another attempts to provide meaning for the same process by making use of visual aids combined with analogy.[6] The third study concerned the use of folded papers to visualize the parts of the whole which are being considered in each problem.[7]

Need for the Study

There appears to be a need for further study concerning division of fractions because of several factors. First, there is not agreement among the authors of arithmetic textbooks as to the preferred method for teaching this skill. Among the nine textbooks observed by the writers, none presented division of fractions by the common denominator method alone, six presented it by the inversion method alone, and three presented it both ways.

Second, it is disturbing to find statements advising teachers to present this skill without meaning, such as that expressed by Brueckner and Grossnickle:

> "It is apparent that the mathematical reason for inverting the divisor is undoubtedly beyond the comprehension of most pupils in the elementary school. No teacher should attempt to rationalize the process with an average class. . . . For that reason, the pupil should be taught to invert the divisor and multiply even though he may not understand the mathematical basis of the operation."[8]

Since the degree of skill attained by the two methods tested appears to be relatively equal, it would seem to be preferable to use the meaningful approach if retention is at least equal, or possibly even better with the common denominator method.

Terms Used

The two terms used, "common denominator method" and "inversion method" may be defined as follows:

The common denominator method of division of fractions involves changing, if necessary, both the divisor and the dividend to a common denominator. The numerators may then be divided to obtain the quotient, as shown below.

$1 \div 1/4 =$	$2/3 \div 1/2 =$	$5/8 \div 2 =$
$4/4 \div 1/4 =$	$4/6 \div 3/6 =$	$5/8 \div 16/8 =$
$4 \div 1 = 4$	$4 \div 3 = 1\ 1/3$	$5 \div 16 = 5/16$
$2\ 1/2 \div 1/2 =$	$1/2 \div 3\ 1/2 =$	$2/5 \div 5 =$
$5/2 \div 1/2 =$	$1/2 \div 7/2 =$	$2/5 \div 25/5 =$
$5 \div 1 = 5$	$1 \div 7 = 1/7$	$2 \div 25 = 2/25$

The main value of this method is that it may be demonstrated visually with chalk board drawings or flannel board figures. For example:

1. First, place on the flannel board three full circles:
2. Write on the chalk board the problem: $3 \div \frac{1}{4} =$
3. Discuss with the pupils the meaning of the division process, being sure to indicate that in this problem we really want to know how many one-fourths there are in three.
4. Next, on the flannel board, replace each of the full circles with four one-fourths, making sure the pupils understand the procedure.
5. At this time, have the class count the number of one-fourths appearing on the flannel board, and rewrite the problem on the chalk board, changing 3 to 12/4: $12/4 \div \frac{1}{4} =$
6. Re-phrase the statement of the numerical problem as indicated in step 3 above asking "How many one-fourth circles are there in three whole circles?" Having just counted these, the answer should be easily understood by the pupils.

[5] Sam Duker, "Rationalizing Division of Fractions," THE ARITHMETIC TEACHER, 1 (1954), 20–23.

[6] Edwin Eagle, "Don't Let that Divisor Become Mysterious," THE ARITHMETIC TEACHER, 1 (1954), 15–17.

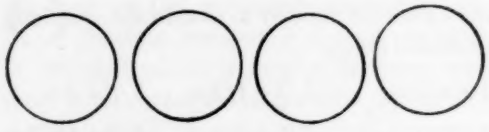
[7] Ann Lawler, foreword by Caroline Hatton Clark, "Picturing Fractions," *The Mathematics Teacher*, 40 (1947), 385–386.

[8] Leo J. Brueckner and Foster E. Grossnickle, Making Arithmetic Meaningful, (Philadelphia: The John C. Winston Company, 1953), p. 338.

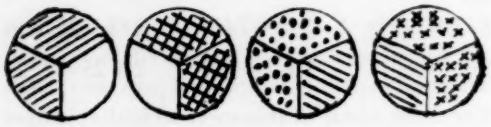
7. Write the answer on the chalk board: 12. Say, "There are 12 one-fourth circles in three whole circles."

In examples in which the divisor is not a unit fraction, the same method can be used. However, making use of color in this procedure may prove helpful, using different colors for each two-thirds:

$4 \div 2/3$



$12/3 \div 2/3$



$12 \div 2 = 6$

The inversion method involves inverting the divisor, then multiplying, as below:

$1 \div 1/4 =$	$2/3 \div 1/2 =$
$1 \times 4/1 = 4$	$2/3 \times 2/1 = 4/3 = 1\ 1/3$
$5/8 \div 2 =$	$2\ 1/2 \div 1/2 =$
$5/8 \times 1/2 = 5/16$	$5/2 \times 5/1 = 10/2 = 5$
$1/2 \div 3\ 1/2 =$	$2/5 \div 5 =$
$1/2 \div 7/2 =$	$2/5 \times 1/5 \div 2/25$

$1/2 \times 2/7 = 2/14 = 1/7$

The difficulty with this method is its inability to be explained or visualized at a level which is understandable to a sixth grade pupil. The rationale for the procedure is based on two facts: (1) Any number multiplied by its reciprocal equals 1. (2) Any fraction may have both its numerator and its denominator multiplied by the same number without changing its value.

For example, consider the problem $\frac{2}{3} \div \frac{1}{2} =$. First, express the problem as a fraction:

$$\frac{2/3}{1/2}$$

Next, multiply both the numerator and the denominator by the reciprocal of the denominator:

$$\frac{2/3 \times 2/1}{1/2 \times 2/1}$$

Performing this operation, we now have:

$$\frac{2/3 \times 2/1}{1}$$

and since the denominator is 1, it may be dropped, leaving the problem, $\frac{2}{3} \times \frac{2}{1} = ?$

Assumptions

In order to consider the results of this study it is necessary to make certain basic assumptions. First, that all students involved in the study were in attendance when division of fractions was taught, or that the teachers gave sufficient individual help to those who were absent. Second, that all teachers in the control group gave equivalent emphasis to the pages of the textbook dealing with division of fractions, and taught this skill as outlined in the teacher's guide. Third, that all teachers of the experimental group understood the common denominator method thoroughly and taught division of fractions rather completely by this method before presenting the inversion method used in the textbook.

Data

The I.Q. scores of the students were obtained from the California Test of Mental Maturity administered in May, 1958, as a part of the regular testing program of the Oxnard School District.

The division of fractions test on which the comparative retention is based was prepared by the writers. It is composed of five word problems and ten computational problems involving the different combinations of

fraction, mixed number, and whole number. The word problems are adaptations of those appearing in textbooks. The same test was administered at the end of the experimental period in May, 1958, and on the first day of school in September, 1958.

For the purpose of measuring the comparative retention of the skill in division of fractions, matched pairs of individuals were chosen, one from the experimental group and one from the control group making up each pair. These students were matched first on the basis of scores on the division of fractions test given in May, and they were further matched on a basis of I.Q. The scores on the division of fractions test were identical in all cases, and the I.Q. scores compared as shown below:

DIFFERENCE IN I.Q.	NUMBER OF PAIRS	% OF TOTAL PAIRS
0–3 points	53	70.7%
4–7 points	13	17.3%
8–13 points	8	12 %

Analysis of Results

The specific question under study was to determine whether or not the skill of division of fractions was retained better over a period of time with no review when taught by the common denominator method or the inversion method. In order to measure this retention, the same test was administered to both the experimental and the control groups near the end of the school term, May, 1958, and again on the first day of school in September, 1958. On a basis of the original test, plus I.Q. scores, matched pairs of students were chosen, and their scores were compared with scores on the test administered in September, 1958.

The two groups of matched pairs had mean scores of 9.5 and medians of 9 on the test given in May, 1958.

The post-test was given at the beginning of the school year, September, 1958. For the experimental group the mean score on this test was 4.9 and the median was 3.5, while for the control group the mean score was 5.5 and the median 3.45.

The sum of the differences between the scores was -27. The standard deviation of the difference between the scores was computed:

$$\sum d^2 = 1235 - \frac{(27)^2}{75}$$

$$\sum d^2 = 1225 \cdot 28$$

$$Sd = \sqrt{\frac{\sum d^2}{n-1}}$$

$$Sd = \sqrt{\frac{1225 \cdot 78}{74}}$$

$$Sd = 4.04$$

The standard error of the difference for the paired observations then is:

$$S\bar{x}_1 - \bar{x}_2 = \frac{Sd}{\sqrt{n}}$$

$$S\bar{x}_1 - \bar{x}_2 = \frac{4.04}{\sqrt{75}}$$

$$S\bar{x}_1 - \bar{x}_2 = .47$$

From this figure the value of t becomes:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S\bar{x}_1 - \bar{x}_2}$$

$$t = \frac{5.17 - 5.40}{.47}$$

$$t = -.49$$

This value, with 74 degrees of freedom then, shows there is not a significant difference between the retention of the skill in division of fractions with these two groups of students at the 5% level.

Conclusions

From these results we may conclude, then, that either method is acceptable, and perhaps both should be taught in order that the process may be given meaning through the use of the common denominator method, and the students may then choose the method with which they work more easily.

Division is Understandable

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SOMETIMES IN OUR TEACHING we fail to build upon children's past experiences to make arithmetical concepts meaningful. Most of us accept the fact that the reading of the printed word is successful if a child has a background of experience to bring to word symbols. We know that a word such as mother has meaning for the child. He can call upon his background of experience to give meaning to this abstract symbol because he has a knowledge of, a feeling for, an attitude about a mother. When he reads the word he draws upon his past experiences to gain meaning. As teachers we capitalize upon this understanding and utilize it in helping the child learn to read.

We are a little reluctant to use this same approach when trying to have children develop an understanding of the number symbols or number processes. Just as word symbols are a means of conveying ideas, so numbers are symbols which convey ideas. Children have backgrounds of experience which they bring to number symbols just as their background of experiences bring meaning to word symbols. As teachers we would be wise to use and build upon the understandings children already have when we introduce them to new facts or processes in arithmetic.

One arithmetical process which has caused children much difficulty is that of division, Yet children have been dividing things almost from the time that they started associating with others. Let us take a look at John's past experiences with the division process. He has two pieces of candy. He gives his friend one and keeps one for himself. A few years later in his life we find John with seven baseball cards. He decides to share with Joe. He gives one card to Joe. He keeps one for himself. Another card is given to Joe. Another card John keeps for himself. This continues until both children have three cards. One card is left over. Since the cards were John's he kept the extra one. John has been using division. He has divided a number of things into a given number of parts. In this case the division made the parts as equal as possible.

John uses division in another situation when he and twelve boys are playing in the vacant lot. They decide to play ball and proceed to choose teams. Each child takes his place behind the captain who has chosen him. The group of boys is divided into two equal groups of six.

In the daily living of the family children find many opportunities to divide things. The family is going on a picnic. The children have to help mother get ready. John helps with the sandwiches. How many can they have? Mother reports that they have twenty slices of bread. One sandwich takes two slices. How many slices of two are there in twenty? John may count off: two, two, two until he has grouped ten sandwiches or pairs. This is repeated subtraction or division. John grouped a number into a given number of parts or measured off to see how many two's there were.

In their play, work, and living children have many similar experiences which cause them to think out the division process. They are using the division process which later will be taught to them as a rather abstract concept.

Division in School

Now let us take a look at the present day method of teaching division. The following anecdote reveals what many fourth grade children are doing with long division in school.

John, who had little difficulty dividing his candy and baseball cards with his friends, supervising the division of a group of boys to form teams, or helping his Mother estimate how many sandwiches they could make, was having a great deal of trouble

doing his arithmetic homework. He called to his father to come upstairs and help him as he was having trouble. His father called to ask what kind of problems John was doing. "Gozunto problems," shouted John.

This is a fictional anecdote but too many children use the expression, "gozunto" and have just as much understanding of what they are doing as the expression indicates. Division to them means one number goes into another. The expression "two goes into eight" has no meaning for many children and brings confusion about a process they have been using in their daily living for a long time.

Another aspect of the problem seems to be that in the schools division is taught as an unrelated, brand-new, mechanistic concept to be learned by rote. Children are told that to divide we use much of the arithmetic which we already know such as multiplication, addition and subtraction. There are five steps one uses in division. These are usually kept before the class on a chart or the blackboard: (1) divide, (2) multiply, (3) subtract, (4) compare, (5) bring down the next number, and then repeat the procedure until all the numbers are used up.

Children taught these steps in a rote-like manner rarely can explain why they use the steps. When questioned about why they used a certain step or asked to explain their solutions their answers are: "because you do," "the teacher told us," "that is the way to divide," and similar meaningless responses.

An approach which is quite the opposite to the approach just described, utilizes the understanding of division children have and furthers this understanding by solving problems which are taken from the child's own experiences. These may be real or imaginary but should be solved orally so that the teacher may follow the children's thinking and check their understanding. The children will bring to class such problems as the one John has. John's problem is to find out how many sandwiches could be made for the gang's Halloween party if he had two loaves of bread of twelve slices each. One child might say that he would take away two from the twenty-four until all the bread was used. After many such experiences this child certainly would realize or could be led to see that division was a series of repeated subtractions.

This situation happens frequently in the modern school where trips are used as a source of information. The class of twenty-three is going on a bus trip. The seats in the bus hold two people. The members of the class will each choose a partner. How many seats in the bus will they use? The problem is solved by acting out the solution with the children or on the flannel board by using representative materials for the children. The demonstrated conclusion is that twenty-three would be eleven groups of two with one left over. That person could join a group of two, sit with the teacher, sit by himself, or whatever the class or he decided. This is needed division and the remainder of one is seen as a vital statistic rather than just something that is left over, the terminology too frequently given to the remainder in a division example. Too, later on when it comes to writing this information, remainders will be written in a descriptive fashion. When a child needs to record a remainder, he will do one of two things; if the symbol represents something that can be further divided, such as money, cookies, and the like, the child will do this further division and leave the number in fraction form; if the symbol represents something that can not be further divided such as the child in this case, the answer will be recorded as a remainder of one.

Many varied but similar problems are solved orally by the children before writing the final arithmetical algorism. The recording of ideas involved in the solution help children to see that what they write down is just what they are thinking. This makes meaningful the steps involved in the division process.

The child who expressed the solution of a problem with this actual demonstration and oral description demonstrated an understanding of the division process.

I have $5.98. I want to divide the money equally among my 4 friends and myself. Each child gets one dollar. I "divvied-out" a dollar to each.

$1.00 each received
5 children ⟌$5.98

$1.00 each received
5 children ⟌$5.98
I must see how much I have used up in that division.
$1.00 five times is $5.00. → 5.00 money used so far
.98
I now have 98 cents. → .98

I can give each person a dime → $ .10 each gets (2nd division)
because I have 9 dimes. 1.00 each gets (1st division)
5 children ⟌$5.98
5.00
.98

$ .10 each gets (2nd division)
1.00 each gets (1st division)
5 children ⟌$5.98
5.00
.98
One dime 5 times is 50 cents. → .50
Forty-eight cents is left over → .48
This means I could have 4 dimes and 8 pennies
Not enough dimes to give each person another dime.
I could give them 9 pennies → 9 each gets (3rd division)
10 each gets (4th division)
$1.00 each gets (1st division)
5 children ⟌$5.98
5.00
.98
.50
.48
9 pennies five times is 45 → .45
3
3 pennies are left I will keep them. → 3

In our teaching of division we will find that if we think of teaching and learning as much more closely related, we will guide the learning of children by using their past experiences and provide opportunities for the children to question, to explore, to experiment, to think things through. Guiding arithmetic learning in this manner will cause children to think of division or any other arithmetic process as one more way of recording their thoughts or ideas. They record on paper the things that they do in daily living and communicate these activities to someone else.

EDITOR'S NOTE. Yes, division is understandable both at the early stages as illustrated by Mrs. Izzo and also at the later stages when the exercises involve larger numbers. There are two kinds of understanding: (a) the significance of the operation as applied to a situation and (b) the process or work with the numbers involved. Note that the pupil whose work has been reproduced worked by a "step analysis method" which has meaning for him at this stage. Later he will reduce the steps as he learns how to think with larger units. But this approach is very different from a direct introduction to the adult pattern of division. It is our job as teachers to guide learning in accord with the level and stage of development of the pupil. It is a mistake to dwell too long on an immature pattern when a higher level can be reached.

Take a Number and Build a Number System

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THE DECIMAL SYSTEM of notation is one of man's greatest inventions. The greatness of the invention is not in the selection of ten as the base, but rather in the *system* that is functional and simple—a system that works with any base that one may wish to use. There are, without doubt, a number of bases that would have been superior to ten.[1] Actually, the origin of ten as a base of our decimal system is not known. Some people state that ten was selected because man has ten fingers, but obviously such a statement oversimplifies the genius that operated in the selection of zero as the initial symbol for the decimal system of numerals.

The ideas of establishing zero as the first or beginning digit in the series of ten digits, of assigning each primary digit a specific value in the series, and utilizing the concept of positional value are indicative of thinking that cannot be termed purely accidental. Let us respect our forefathers, and especially let this respect grow into appreciation for ideas that facilitate simple communication and computation. Contrast the ease and simplicity of finding the product of 255×25 and of dividing 486 by 6 using decimal notation with that of Roman notation!

Different number bases have been used by people of ancient civilizations. The ancient Greeks used twenty "digits," perhaps because they wore open-toed sandals and their toes were readily available for counting. The Babylonians based their numbers on a base of sixty. Even today there are vestiges of this number system—sixty minutes in one hour, sixty seconds in one minute. Even modern languages have words that are reminiscent of non-decimal number systems. For example, the French word for eighty is "quatre-vingt"—five twenties.

Non-decimal number systems are of special interest today in the computer field. Electronic computers to arithmetic mainly using a binary system. The binary system is based on only two digits, 0 and 1. Arithmetic based on only two digits is well suited to computer work, for it is simple to manipulate 0's and 1's with switches that are either on or off, or with cores that are magnetized in either of two directions.

Let us note some of the principles of the system using decimal notation and then apply these same principles to some bases other than ten.

The principle of representing numbers in the binary system is the same as the principle for decimal representation. For example in the decimal system 372 means

$$3 \text{ hundreds} + 7 \text{ tens} + 2 \text{ ones} = 3 \times 10 \times 10 + 7 \times 10 + 2$$

or, using exponents,

$$3 \times 10^2 + 7 \times 10^1 + 2 \times 10^0.$$

(In exponential notation $10^0 = 1$, $10^1 = 10$, $10^2 = 10 \times 10 = 100$, $10^3 = 10 \times 10 \times 10 = 1000$, etc.)

Similarly, in the binary system 10101 stands for

$$1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$
$$= 1 \times 2 \times 2 \times 2 \times 2 + 0 \times 2 \times 2 \times 2 + 1 \times 2 \times 2 + 0 \times 2 + 1.$$

($2^4 = 2 \times 2 \times 2 \times 2 = 16$, $2^3 = 2 \times 2 \times 2 = 8$, $2^2 = 2 \times 2 = 4$, $2^1 = 2$, $2^0 = 1$).

Adding up these terms shows that the symbol 10101 in the binary system has the same value as the symbol 21 in the decimal system.

[1] The Duodecimal Society of America has become the center for research and publication in the field of proposing changing our counting to the base twelve. It issues the *Duodecimal Bulletin*.

The rules for adding and multiplying numbers in the binary system are very simple. The addition table for the binary system is shown below.

Top Addend

Lower Addend	+	0	1
	0	0	1
	1	1	10

The symbol 10 in the binary system means one two and no ones.

The multiplication table for the binary system is equally as simple.

Multiplicand

Multiplier	×	0	1
	0	0	0
	1	0	1

Let us now take any number and build a number system using the principles of our familiar decimal system. Since any base number would work just as well as the bases ten (decimal) or two (binary), let us take the number five as a base. Using five as a base you need five digits to write numbers. In this system the digits in a number symbol represent multiples of powers of five. For example:

$$123 = 1\times5^2+2\times5^1+3\times5^0$$

$$3041 = 3\times5^3+0\times5^2+4\times5^1+1\times5^0.$$

Thus 123 in the base five system has the same value as 38 in the decimal system.

$$(123 = 1\times5^2+2\times5^1+3\times5^0$$
$$= 25+10+3 = 38.)$$

The addition table for the base five system is given below.

Upper Addend

Lower Addend	+	0	1	2	3	4
	0	0	1	2	3	4
	1	1	2	3	4	10
	2	2	3	4	10	11
	3	3	4	10	11	12
	4	4	10	11	12	13

The multiplication table for base five is

Multiplicand

Multiplier	×	0	1	2	3	4
	0	0	0	0	0	0
	1	0	1	2	3	4
	2	0	2	4	11	13
	3	0	3	11	14	22
	4	0	4	13	22	31

Using the multiplication and addition facts given in these tables, here is an example in multiplication:

Base Five	Same Number in Base Ten (Decimal)
$234 = 2\times5^2+3\times5^1+4\times5^0$	= 69
$23 = 2\times5^1+3\times5^0$	= 13
1312	207
1023	69

$$12042 = 1\times5^4+2\times5^3+0\times5^2+4\times5^1+2\times5^0 = 897$$

You can take any number to use as a base and make a number system. Of course, if you use a base larger than ten, you need new digits. For example, if you use the number twelve as a base (duodecimal system) you need twelve digits to write numbers in your system. Using x for ten and E for eleven, your digits in the duodecimal system would be

0 1 2 3 4 5 6 7 8 9 x E.

The symbol 10 in the duodecimal system means $1\times12^1+0\times12^0$ or one dozen. The number xE in the duodecimal system means ten twelves and eleven ones or 131 in the decimal system.

The higher the base number you use in constructing your number system, the fewer digits it takes to represent any given number. For example, the year 1960 represented in various number systems is

Binary	11110101000
Base Five	32320
Base Eight	3650
Duodecimal	1174

Using the base sixty, the number 1960 could be written using only two digits.

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EDITOR'S NOTE. In recent years many people have used a base other than ten to illustrate the principles and structure of our Hindu-Arabic number system. This system is certainly one of man's greatest achievements. As Dr. Willerding points out, its basic principles can be applied to any base. Electronic computers use base two because that fits their limitations. A base of eight would be suitable for many common fractions when converted to "point notation" and a base twelve would serve well with measures that have a similar unit of conversion. If by some evil stroke of magic one could arbitrarily convert the base of our monetary system from ten to two then $1960 (base 10) would yield $11,110,101,000 (base 2) which looks like a great deal of money. Inflation has not yet reached these proportions.

On Labeling Answers

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Herzl Elementary School, Chicago, Ill.

WHEN THE CONDITIONS or units in arithmetic problems are stated should teachers insist that students label their answers accurately? To make the case clear, consider the problems in the chart at the bottom of the page and the various ways in which students might write their answers.

From my point of view an accurate label for the answer to each problem is essential. For it proves that the student, upon reading the problem, understood precisely what was to be found. It exemplifies respect for clarity and finish—two fundamental traits of good performance in mathematics. And it supplies a useful reminder of the conditions of the problem for readers of the student's paper, or for the student himself when corrections are necessary.

PROBLEM	ANSWERS		
	Inadequate or not clear	Adequate but not precise	Accurate and concise
$2.50×4	1,000 $1.000	10.00	$10.00
½ of 36 in. = ?	18 18 ft.	18 in. 18 inch	18 in. 1½ ft. ½ yard
Find the area of a floor 15 feet wide and 20 feet long.	300 300 ft.		300 sq. ft.
The children bought 25 bags of candy, 10 packages of gum, and 5 pencil cases to be used as prizes. How many prizes were there in all?	40 40 bags	40 in all 40 candy, gum and pencils	40 prizes

On the other hand, having students write full statements to answer the questions of arithmetic problems certainly is not essential. I oppose that practice. The purpose of labeling answers is simply to specify what the answers represent. The more directly this is accomplished, the better. For is any virtue of mathematics more conducive to appreciating its beauty than *brevity* without loss of accuracy?

The problem of getting students to label answers accurately and *habitually* is centered, then, in awakening their imaginations to mathematical finesse, in much the same way in which their imaginations are naturally awakened to an expert play in a game of baseball, or to a neat skip in a fast double-dutch rope. It is delight that so awakens and conquers—delight in the clarity and economy of means, delight in the precision of results. By the glow of our enthusiasm for it, by the strength of our persistence in having them sample it, by the sincerity of our praise of their success in doing so, we ought to help students discover this delight. Insisting on their labeling answers to arithmetic problems accurately is an easy point from which to start this important task.

True: this requirement, like any others of this sort, must be relaxed in a few unique cases—for children of low ability or for the early stages of learning a new skill. But a large majority of the students in any class will eventually take pride in meeting it if it is emphasized with *intelligent* persistence.

EDITOR'S NOTE. The editor agrees wholeheartedly with the author that there is considerable value in taking pride in one's work and in the form of presentation of an answer. There is now wide variation of practice in respect to credits to be assigned to answers to problems where a label is expected. Consider Mr. Clendenon's last problem. Some will argue that the answer "40" is perfectly correct and should receive full credit. Others will give such an answer only 7 or 8 out of total of 10 possible points. Should all pupils be treated alike? The boy with high I.Q. who easily gets all the correct numbers for answers but fails to provide labels may feel unjustly penalized when discounted.

Can someone state general principles or policies? For example, if a problem asks or implies the unit or label of an answer then it should be given full credit if the label is missing but if the answer depends upon a label for correctness and this label is missing, then it is appropriate to give less than full credit. To illustrate, if a problem asks for the number of square feet of area of a rectangle then an answer of 120 should be considered correct but if it asks for the area then a label such as "sq. ft." must be included with the answer. Is this too broad? Suppose a label is misspelled or an abbreviation is incorrect? Most teachers will agree that we should hold a high ideal.

Reviews of Books and Materials

Edited by E. Glenadine Gibb

The Math Entertainer, Philip Heafford. New York: Emerson Books, Inc., 1959. 176 pp., $2.95.

The Math Entertainer, a mathematical quiz book, is designed to puzzle, delight, and entertain its readers. The subject matter of the fifty quizzes covers a wide assortment of topics from arithmetic, algebra, geometry, trigonometry, calculus, and statistics. Each quiz is a short adventure into some mathematical area. Each adventure is different in subject matter and difficulty.

Due to the great diversity in subject matter and difficulty, the questions contained in the quizzes might be more exasperating than entertaining. However, each question is clarified and resolved by unusally complete and instructive explanations that are found in a uniquely colored page section in which the daft problem-solver can easily regain his composure.

Other than a few slight errors, such as printing 3.1416 . . . to represent π, the book as a whole appears to be in good form.

Since the quizzes are full of serious mathematics, the 176 page book should be an instructive aid to any mathematics club or mathematics teacher.

The title, *The Math Entertainer*, is accurate in that the contents of the book will be entertaining to anyone with a slight interest in mathematics—*Jens A. Jensen, Iowa State Teachers College*

Learning to Use Arithmetic Series, Grade 7 (teacher editions, pupil textbooks, workbooks), Agnes G. Gunderson, George E. Hollister, Joseph H. Randall, Joseph J. Urbancek, F. Lynwood Wren, J. Wayne Wrightstone. Boston: D. C. Heath and Company, 1959.

The 1959 edition of *Learning To Use Arithmetic, Grade 7* offers a rather standard arithmetic program packaged in an attractive format. The authors' philosophy and the learning experiences advocated throughout the book, manual and workbook seem to implement both the social and the mathematical aims of arithmetic instruction.

Teacher's Edition. The seventh grade teacher's edition is a looseleaf, hard cover, ring binder. This unique manual contains a page of the teacher's guide for every page of text. The philosophy of flexibility, which the construction of this manual could engender, is most noteworthy.

A very helpful feature of the manual is a section preceding each chapter called, "About Chapter ———." These sections inform the teacher as to what is in each chapter and why. This should be very useful to the teacher for purposes of long range planning. In this page-by-page manual, there are instructions for using the textbook, evaluation procedures, related activities, and suggestions for handling individual differences. An important feature of this manual is that it provides many suggestions but does not attempt to be "all-inclusive." Teachers are free to adapt, implement, improvise and select learning experiences for all children.

Book Seven. In this book the authors have attempted to present an experience-centered, developmental program. Mathematical concepts are presented through interesting, up-to-date problem material and effective use of visual aids. There is an abundance of practice materials strategically placed throughout the text. For pupils at all levels, enrichment materials are offered at spaced intervals throughout the book. A protractor and ruler with three different scales is included in the back of each book.

This book follows the usual pattern of topics offered as the arithmetic program for seventh grade. Most people will agree that review, reteaching and maintenance are essential to any good arithmetic program. However, there is a question in the minds of

many arithmetic authorities as to the value of such heavy emphasis upon what has previously been taught in the other six grades. With such an emphasis on previously presented concepts of whole numbers, fractions, decimals and measurement, little room is left for expansion of beginning algebraic concepts, geometry, graphs, history and development of the number system, and other relatively new and stimulating topics.

The authors should be congratulated for their excellent section entitled, "Understanding Graphs." Their unique use of clear-cut diagrams and color, fairly makes the concepts concerning graphs "come alive." The inclusion of resource tables and a glossary of quantitative terminology at the back of the book is a most desirable feature. As with the rest of the series, continued emphasis is placed upon developing skills of estimation, approximation, and mental arithmetic.

Workbook for Grade Seven. This paperbound workbook is correlated with the textbook. A teacher's edition of the workbook with written-in answers is provided. The format is clean and uncluttered. Color has been judiciously used. As with all workbooks so closely related to the textbook, there is danger that they will be used page by page with every child on the same page at the same time. On the other hand, judicious use of this workbook could provide individual children with additional learning experiences and more practice materials.

D. C. Heath's, *Learning To Use Arithmetic, Grade Seven* should enable children to further develop their understanding and appreciation of arithmetic. Although the book presents a rather standard program, the up-to-date problem settings and the excellent use of visuals should enable teachers to guide children to use their own mathematical reasoning powers.—*Ronald C. Welch, Indiana University*

Seeing Through Arithmetic Series, Grades 3, 4, 5, 6, Maurice L. Hartung, Henry Van Engen, Lois Knowles. Chicago: Scott, Foresman and Company, 1957–1959.

These books were examined by a group of teachers whose collective experience included teaching at all grade levels in the elementary and secondary school. The criteria for the evaluation were expressed in the following questions; Are the contents mathematically sound? Will the books be usable and appealing to the pupil? Are the methods of presentation based on sound learning theory? Will the books help the teacher to present meaningful lessons? Will use of the books result in broad mathematical understandings which utilize earlier number experiences of the child and prepare him for mathematical experiences in the secondary school? Features of the series relevant to these questions are discussed below.

The format is excellent, the books being of convenient handling size and expertly bound so they will open and lie flat without straining the binding. The pages are attractively and colorfully illustrated. Each picture is a visual aid to the lesson, with no pictures presented for decorative or interest-gathering values. The illustrations concerning plane measurement in the fifth grade book are a good example of this feature. The type is clear and readable, with form and size varied to fit the particular items presented.

The contents of the books are organized under headings such as "Learning How," "Keeping Skillful," "Exploring Problems," etc. Not only the teacher, but also the child, will always recognize whether he is learning new ideas, practicing new skills, maintaining formerly developed skills, or applying skills in problem situations. Each lesson is preceded by a simple statement of the goal of the lesson and concludes with a summary statement of what has been learned. Each lesson presenting new ideas is divided into sections called "See," "Think," "Try," and "Do," where the child sees what happens in a problem situation, thinks symbolically about what happens, tries this thinking in another situation, and finally does some exercises to see if he can use this thinking in situations where he is not aided by the textbook illustrations.

The vocabulary of books has been carefully controlled. For each lesson the teaching guide lists all words which will be new to the children. Unlike many arithmetic books, page space is not used for detailed explanations of operations. The authors seem to assume that these explanations are not necessary, that the child will discover these for himself as a result of experiences planned and guided by the teacher.

One feature of the series which will probably cause much discussion is the use of the "place-holder" in "equations" at all levels. The child first learns to record what happens in a simple mathematical sentence. A "number screen" is placed in the sentence to hold a place for the element which is sought. The child is led intuitively to discover means of finding the numeral which is "hiding behind the number screen." He is never taught mechanically to solve the equation. Teachers of secondary mathematics should appreciate receiving students from elementary school with this understanding of the function of a "place-holder." In Grade Five the number screen is replaced by the place holder "n."

A feature of the series which may excite some controversy is its treatment of division. Throughout the series division is presented as a process of repeated subtraction. Some teachers may accept this treatment with misgivings. However, a study of its development in the series shows that it has real advantages; it can be developed easily by use of concrete materials, the number of digits in the divisor becomes unimportant, difficulties with zeroes in dividend and quotient are eliminated, and its use is flexible enough to allow for wide individual differences among students.

The last book in the series contains some material which is not usually found in a sixth grade book (i.e., a lesson on inequalities). This should appeal to the student and contribute to the continuing process of building an understanding of the over-all structure of mathematics. The solution of per cent problems by the use of pairs of equal ratios is introduced in the fifth grade book and extended in the sixth grade book. This treatment of per cent is logical, easily understandable, and mathematically sound. The authors have included exercises involving all three possible per cent situations. Normally this is deferred until grade seven or grade eight. The reviewers feel that the ratio treatment simplifies the teaching of per cent so much that it can be taught effectively in grade six.

The teaching guides for this series are thoroughly and expertly done. "Lesson Briefs" provide reproductions of pupil lesson pages, objectives for lessons, lists of new vocabulary, and and an answer key for exercises. "Expanded Notes" give more detailed suggestions for teaching each lesson, and they include teaching suggestions for the class as a whole, the able pupil, and the slow learner. The "Activities" section is a real source book of games and activities useful in arithmetic. Wisely used, these guides should provide a great deal of help in planning lessons and suggest things the creative teacher might find useful for any particular group of pupils.

The reviewers found *Seeing Through Arithmetic* to be an excellently written series, attractive and highly usable for both teacher and pupil. The content, organization, illustration, and methods of presentation seem to be based on a thorough knowledge of principles of learning. Use of these books should result in meaningful lessons which develop a real understanding of our number system and operations within that system.—*Della McMahon, Max Hosier, Ruth Hutcheson,* and *Eleanor McBride.*

Educational Motion Pictures

The Audo Visual Center of Indiana State University at Bloomington has issued its 1960 catalog which lists 5784 titles of educational films which were prepared by various producers. In this listing there are perhaps thirty that are useful for teaching arithmetic in the elementary school. Titles range from "Addition is Easy" to "What is Four?" alphabetically and within this range practically every topic of arithmetic is listed.

NCTM

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Report of the Nominating Committee

The Committee on Nominations and Elections presents its slate of nominees for offices to be filled in the 1960 election. The term of office for the president and the two vice-presidents is two years. Three directors are to be elected for terms of three years.

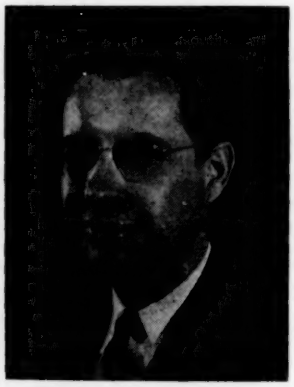
In making nominations for the three director positions, the Committee followed the directive adopted by the Board of Directors in 1955 which states, "Nominations shall be made so that there shall be not more than one director elected from each state, and that there shall be one director, and not more than two, elected from each region." Members may consult THE MATHEMATICS TEACHER for October, 1955, for a map of the regions as they are now defined.

Ballots will be mailed on or before February 26, 1960 from the Washington Office to members of record as of that date. Ballots returned and postmarked not later than March 26, 1960 will be counted.

The Committee wishes to thank the many members of the NCTM for help in giving their suggestions for nominees. It is hoped that all members of our organization will be sure to exercise their privilege of voting.

IDA BERNHARD PUETT, *Chairman*
MILTON BECKMANN
CHARLES BUTLER
HOWARD F. FEHR
KENNETH B. HENDERSON
HOUSTON T. KARNES
ALBERT LINTON
ANN PETERS
OSCAR SCHAAF

NOMINEES FOR PRESIDENT



PHILLIP S. JONES

HENRY VAN ENGEN

Phillip S. Jones

Professor of Mathematics and Education, University of Michigan, Ann Arbor.

A.B., M.A., Ph.D., University of Michigan.

Jackson High School, Jackson Junior College, Jackson, Mich.; Edison Institute of Technology, Dearborn, Mich.; University School, Ohio State University; Visiting Associate Professor, University of California; instr., Summer Inst., Duke Univ.

Member: NCTM; AMS; MAA; Phi Rho Pi; Sigma Xi; Phi Kappa Phi; AAAS, History of Science Society.

Activities in NCTM: Vice-President; member, Board of Directors; Departmental Editor, Associate Editor, THE MATHEMATICS TEACHER; Editor, *24th Yearbook*.

Other activities: member, NAS-NRC Film Committee; SMSG Advisory Committee; Mathematics Committee, CEEB; formerly member of Board of Governors of MAA; Co-operative Committee on the Teaching of Science and Mathematics of the AAAS.

Publications: Articles in THE MATHEMATICS TEACHER, *The American Mathematical Monthly*, and other magazines.

Henry Van Engen

Professor of Education and Mathematics, University of Wisconsin, Madison, Wisconsin.

A.B., Nebraska Wesleyan University; Ph.D., University of Michigan.

Elementary, junior high, and senior high schools, Nebraska, Michigan, and Ohio; Western Reserve University; Kansas State University; Iowa State Teachers College.

Member: NCTM; MAA; AMS; Sigma Xi; Phi Kappa Phi; Phi Beta Kappa; Pi Mu Epsilon; Kappa Mu Epsilon.

Activities in NCTM: Editor, THE MATHEMATICS TEACHER; member, Board of NCTM; member, Elementary Curriculum Committee.

Other activities: Past President, Kappa Mu Epsilon; member, SMSG Advisory Committee, Commission on Mathematics, CEEB.

Publications: Numerous articles appearing in THE MATHEMATICS TEACHER, *School and Society*, *School Science and Mathematics*, *The Arithmetic Teacher*, and other magazines; chapters in various NCTM yearbooks; coauthor of textbooks.

NOMINEES FOR VICE-PRESIDENT—SECONDARY SCHOOL LEVEL

WILLIAM H. GLENN

EUNICE LEWIS

William H. Glenn

Mathematics Supervisor, Pasadena City Schools, Pasadena, California.

A.B., M.A., University of California at Los Angeles; further graduate work at California Institute of Technology, U.C.L.A., and Los Angeles State College.

Mathematics teacher, Pasadena Junior College, 1939–43; physics teacher, Army Specialized Training Program, 1943–44; assistant director, Research and Development Laboratory, A. O. Smith Corporation, Pacific Coast Division, 1944–46; chairman, Natural Science Division, John Muir College, 1946–50; assistant coordinator, Natural Science Division, Pasadena City Schools, 1950–53; Chairman, Mathematics and Astronomy Department, Pasadena City College, 1953–54; Assistant Principal, John Muir High School, Pasadena, 1954–56; Mathematics Supervisor, Pasadena City Schools, 1956–.

Study group leader three different summers at the California Conference for Teachers of Mathematics and member of the planning committee for all nine of the sessions; co-chairman of an arithmetic workshop in the summer of 1957 in Pasadena; chairman of two Pasadena workshops in the summer of 1959, one in elementary arithmetic and one in secondary mathematics; instructor in content and methods class in arithmetic for Pasadena teachers, 1959.

Member: NCTM; NEA; MAA; ASEE; ASCD; ACEI; PDK; CASMT; AVEAC; CTA; CMC; PEA.

Activities in NCTM: California State Representative; member of NCTM Committee on Co-operation with Industry; Associate Editor, *Mathematics Student Journal;* speaker and discussion leader at many NCTM conventions.

Other activities: Secretary, California Mathematics Council, Southern Section, 1951–53; President, California Mathematics Council, 1953–55; Treasurer, California Science Teachers Association, Southern Section, 1951–53; President, Beta Delta Field Chapter, Phi Delta Kappa, 1954; Vice-President, Pasadena Education Association, 1952.

Publications: Articles in the *Twenty-second Yearbook of NCTM*, THE MATHEMATICS TEACHER, *The Arithmetic Teacher, Teaching Tools, The California Mathematics Council Bulletin, The Clearing House,* and *The Bulletin of the National Association of Secondary School Principals.* Mathematics consultant for *Fractions Kit,* Models of Industry. Co-developer of *The Direct-O-Percenter.* Co-ordinator of mathematics publications for the Pasadena City Schools: *Tips to Teachers; General Mathematics Guide; Using the Language of Algebra;* and *Exploratory Algebra.*

Eunice Lewis

Associate Professor at the University of Oklahoma; teacher of mathematics at the University High School and supervisor of mathematics teacher trainees.

B.A. in mathematics, University of Oklahoma; M.A. in mathematics, University of Oklahoma.

Teacher of mathematics, Covington High School, Covington, Oklahoma; Sapulpa High School, Sapulpa, Oklahoma; Tulsa Central High School, Tulsa, Oklahoma; University High School, University of Oklahoma, Norman, Oklahoma.

Activities in NCTM: Oklahoma State Representative for the past 15 years; Southwest Regional Chairman for the affiliated groups.

Member: Oklahoma Council of Teachers of Mathematics, Mathematical Association of America, Oklahoma Education Association, National Education Association, Delta Kappa Gamma, Kappa Delta Pi, Pi Mu Epsilon.

Other activities: Appeared on programs of the National Council of Teachers of Mathematics, and others of national, state, and local scope. Consultant at the Mathematics Institute held at Louisiana State University for three summers; at the Mathematics Institute held at the

University of Oklahoma for two years; at the Mathematics Conference held at Central State College, Edmond, Oklahoma; and at in-service conferences sponsored by schools in Oklahoma. A visiting lecturer at the Mathematics and Science Institute sponsored by the National Science Foundation at the University of Oklahoma, during the summer of 1959; a visiting lecturer at the University of Illinois, 1958–1959, to work with the University of Illinois committee on School Mathematics and with the Arithmetic Project. Member of the School Mathematics Study Group held at Yale during the summer of 1958 and at the University of Colorado during the summer of 1959.

Publications: Several articles published in THE MATHEMATICS TEACHER and other periodicals.

NOMINEES FOR VICE-PRESIDENT—ELEMENTARY SCHOOL LEVEL

JOYCE BENBROOK

CLARENCE ETHEL HARDGROVE

Joyce Benbrook

Professor of Elementary Education, University of Houston, Houston, Texas.

M.A., Ph.D., The University of Texas.

Elementary and junior high school teacher and elementary school principal, Rosenberg, Texas; arithmetic teacher, grades 7 and 8, Teague, Texas; elementary teacher, Houston, Texas; Teaching Fellow, University of Texas.

Member: NCTM; Texas State Teachers Association; Texas Elementary Principals and Supervisors Association; Texas Council of Teachers of Mathematics; Houston Council of Teachers of Mathematics; Phi Beta Kappa; Pi Sigma Alpha; Pi Lambda Theta; Phi Kappa Phi; Kappa Delta Pi; Delta Kappa Gamma.

Activities in NCTM: Member, Elementary Curriculum Committee, 1955–59, Handbook Committee, 1953–55.

Other activities: Member of the organization committee and first President of the Texas Council of Teachers of Mathematics; Chairman, Elementary Curriculum Committee of the Mathematics Study Commission of the Texas Education Agency, 1958–59; Director of the Mathematics Institutes, University of Houston, 1951–55; member of the Louisiana State University Mathematics Institute staff, 1954; member of Baylor University Arithmetic Workshop staff, 1952; consultant in arithmetic for several Texas school systems; speaker at local, district, and state meetings of mathematics teachers; participant, National Science Foundation Institute on the Teaching of Arithmetic, University of Michigan, 1959.

Publications: Coauthor of *Working with Numbers*, worktexts for grades 1, 2, and 3, *Contributions in Arithmetic*, and *Working with Numbers Teaching Aids*.

Clarence Ethel Hardgrove

Professor of Mathematics, Northern Illinois University, DeKalb, Illinois.

A.B., Marshall College, Huntington, W. Va.; M.A., West Virginia University; Ph.D., Ohio State University.

Teacher in the elementary, junior high, and secondary schools of West Virginia; instructor, Ohio State Univ.; Professor of Math., Northern Illinois Univ.

Member: National Council of Teachers of Mathematics; Illinois Council of Teachers of Mathematics; Ohio Council of Teachers of Mathematics; Central Association of Science and Mathematics Teachers; Association for Supervision and Curriculum Development; Association of Childhood Education; Kappa Delta Pi; Pi Lambda Theta; Sigma Zeta; AAUW; AAUP.

Activities: Vice-President, Illinois Council of Teachers of Mathematics; Chairman of Elementary Section, Central Association of Science and Mathematics Teachers; speaker at numerous mathematics meetings of National and Illinois Councils of Teachers of Mathematics and other groups; consultant for curriculum committees, workshops, and institutes; chairman, Illinois Curriculum Program Committee on Mathematics Curriculum of the Elementary School.

Publications: Coauthor, *Thinking in the Language of Mathematics*, Illinois Curriculum Program; articles in *Twenty-second Yearbook* and *Twenty-fifth Yearbook* of the National Council of Teachers of Mathematics; articles in periodicals; Bulletins for Northern Illinois University Bulletin Service.

NOMINEES FOR THE BOARD OF DIRECTORS

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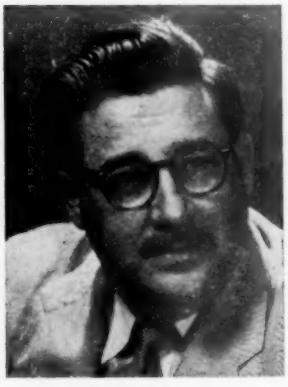

ROBERT E. K. ROURKE

J. Houston Banks

Southeastern Region

Professor of Mathematics, George Peabody College for Teachers.

B.S., Tennessee Polytechnic Institute; M.A. and Ph.D., George Peabody College for Teachers.

Teacher and principal, elementary schools and high schools; Dean and mathematics instructor, East Central Junior College, Decatur, Mississippi; Mathematics Instructor, 133rd Detachment, Army Air Corps; Professor of Mathematics, Florence State College, Alabama; Visiting Professor of Education, Alabama Polytechnic Institute.

Member: NCTM; MAA; NEA; Tennessee Teachers of Mathematics; Tennessee Academy of Science; TEA; Phi Delta Kappa; Kappa Delta Pi; Pi Mu Epsilon.

Activities in NCTM: Member, Committee on Supplementary Publications; Southeastern Representative for Affiliated Groups; Yearbook Planning Committee; former State Representative.

Other activities: Speaker at NCTM and other professional meetings; consultant assignments; guest lecturer, institutes, workshops and conferences; Chief Consultant in Mathematics, statewide N.S.F. summer high school program for the gifted; active in organizing Tennessee Teachers of Mathematics, President 1954–55; active part in organizing and conducting state-wide high school mathematics contests; chairman, committee on high school contests, Southeastern Section, MAA; former President, Mathematics Section, TEA; former State Representative, NCTM. Research in relative effectiveness of various types of college freshman programs, in best utilization of graduate assistants, and in teaching by television.

Publications: Contributor to THE MATHEMATICS TEACHER, *Tennessee Academy of Science Journal;* author, *Mathematics for General Education, Elements of Mathematics, Learning and Teaching Arithmetic.*

Irvin H. Brune

North Central Region

Professor of Mathematics, Iowa State Teachers College, Cedar Falls, Iowa.

B.S., The College of Wooster; M.A., Ph.D., Ohio State University.

Senior high school teacher and principal, Lexington, Ohio; junior and senior high schools, Mansfield, Ohio; Montclair, New Jersey, State Teachers College; Frostburg, Maryland, State Teachers College.

Member: NCTM; MAA; CASMT; AAUP; Iowa Association of Mathematics Teachers; Iowa Academy of Science; Phi Beta Kappa; Phi Delta Kappa; Alpha Delta Omega; Kappa Mu Epsilon.

Activities in NCTM: Assistant Editor, THE MATHEMATICS TEACHER, 1953–59; Editor, *Revised Guidance Pamphlet,* 1953; Chairman, Committee on Guidance Pamphlet, 1959. Member, Conference on Directions, 1959.

Other activities: Advisory Board, IAMT; Chairman, Committee on High School-College Teaching, Iowa Section, MAA.

Publications: articles in THE MATHEMATICS TEACHER, *The Arithmetic Teacher, NEA Journal, Education, Midland Schools, Bulletin of General Semantics, Bulletin of the National Association of Secondary School Principals;* chapter in the *Twenty-first Yearbook,* NCTM.

Eva A. Crangle

Western Region

Teacher-consultant in Mathematics, Junior and Senior High Schools, Salt Lake City Public Schools.

B.S., University of Nebraska; M.A., Columbia University; Shell Merit Fellowship, Leland Stanford University, summer 1956; NSF In-service Institute, University of Utah, 1959–60.

Junior high school mathematics teacher, Bryant Junior High, Salt Lake City;

senior high school mathematics teacher, West High and Highland High, Salt Lake City, Utah.

Member: NCTM; NEA; Utah Council of Teachers of Mathematics; Utah Education Association; Salt Lake City Principals and Supervisors Association; Salt Lake City Teachers Association; Pi Lambda Theta; Kappa Delta Pi; Delta Kappa Gamma.

Activities in NCTM: present State Representative for NCTM in Utah; chairman of Local Arrangements for Twentieth Summer Meeting, 1960; participant on NCTM Convention programs; committee member; convention delegate representing Utah.

Other activities: Member citywide textbook committees; member evaluation team, Kellogg Evaluation Study (under the auspices of the Kellogg Foundation); member Utah State Mathematics Curriculum Committee; participant in annual programs of the Mathematics Section, Utah Education Association conventions.

Publications: Contributor to the Utah Mathematics Course of Study, Grades Seven to Twelve; unpublished supervisory bulletins and outlines for summer school course of study.

Eugene D. Nichols

Southeastern Region

Associate Professor of Mathematics Education, Florida State University, Tallahassee, Florida.

B.S., University of Chicago; M.A. and Ph.D., University of Illinois.

Teacher of Mathematics at University School, University of Illinois; Urbana High School, Urbana, Illinois; Illinois Commercial College, Champaign, Illinois; instructor in Mathematics, Roberts Wesleyan College, N. Chili, New York; Director, 1958 and 1959 NSF Sponsored Summer Mathematics Programs for Talented High School Students.

Member: NCTM; MAA; AMS; Illinois CTM; Florida CTM; Florida Education Association; Phi Delta Kappa; Kappa Delta Pi; Pi Mu Epsilon.

Activities in NCTM: Appearances on convention programs.

Other activities: member, MAA Committee on High School Contests; Florida Director, MAA High School Contest; member, Curriculum Committee, Florida CTM; Editor, *Florida Math Journal;* Past President, Illinois Alpha Chapter of Kappa Delta Pi; consultant to mathematics teacher institutes; mathematics editor for a textbook publisher.

Publications: Coauthor of a chapter in NCTM 24th Yearbook, articles in THE MATHEMATICS TEACHER, *The Mathematics Student Journal, Bulletin of the National Association of Secondary School Principals, The Social Studies.*

Z. L. Loflin

Southwestern Region

Professor and Head, Mathematics Department, Southwestern Louisiana Institute.

B.S., M.S., Louisiana State University; Ph.D., Columbia University.

Member: NCTM; Phi Kappa Phi; Pi Mu Epsilon; Kappa Mu Epsilon; Louisiana Teachers Association; NEA; American Educational Research Association; American Mathematical Association; American Chemical Society; AAUP; American Institute of Mining and Metallurgical Engineers; American Association for Engineering Education; Central Association of Science and Mathematics Teachers; Rotary Club of Lafayette. Listed in *American Men of Science* and *Who's Who in the South and Southwest.*

Activities in NCTM: Editorial Board, THE MATHEMATICS TEACHER.

Other activities: member, National Board of Governors of Mathematical Association of America; past national President, Theta Xi.

Robert E. K. Rourke

Northeastern Region

Head, Department of Mathematics, Kent School, Kent, Connecticut.

Honour B.A., medallist in mathematics, Queen's University, Kingston, Ontario; A.M., Shattuck Scholar, Harvard; course work for doctorate in mathematics, Harvard.

Teacher, Head, Department of Mathematics, Headmaster, Pickering College, Newmarket, Ontario.

Summer School lecturer at Queen's University; University of Alberta; Teachers College, Columbia University. Consultant on mathematics to numerous school systems.

Activities: Past President, Ontario Association of Teachers of Mathematics and Physics; formerly on executive of Canadian Mathematical Congress; director of NCTM 1957–59; Executive Director of the Commission on Mathematics of the CEEB; member of Advisory Council, School Mathematics Study Group; committee of MAA re Survey and Seminar of OEEC.

Publications: Coauthor of *An Advanced Course in Algebra; Trigonometry and Statics; Mathematics for Canadians*, Books I, II, III; articles in professional literature.

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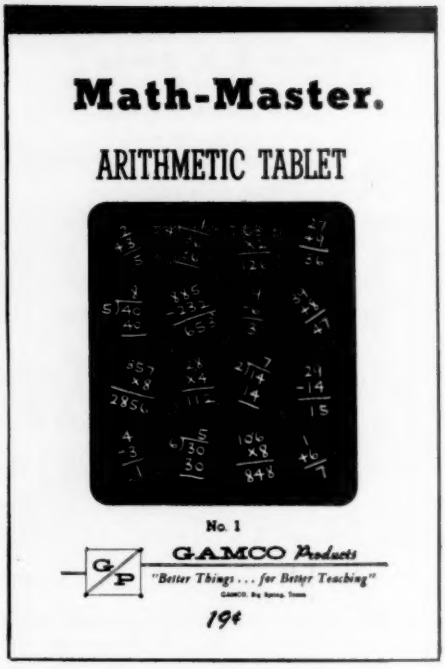

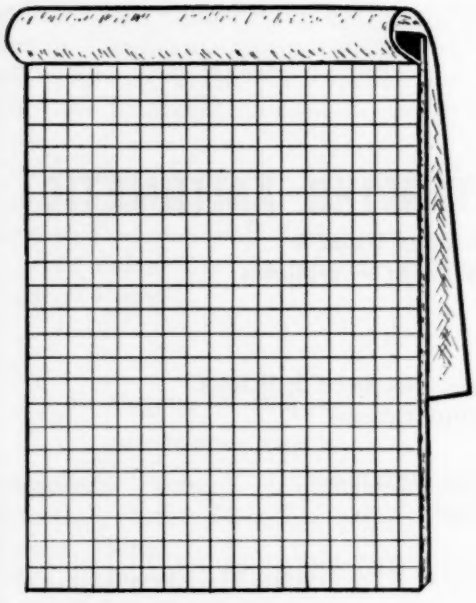

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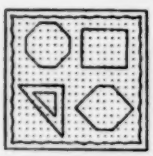

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